Worksheet 5-8: Doubling and Half-Life

Doubling: Doubling time is the time it takes for a population to double in size.

The relation for doubling is $P = P_0(2)^{\frac{t}{d}}$, where P represents the population,
P_0 represents the initial population,
t represents time
d represents the doubling time, and
the base "2" indicates doubling

1. A bacteria culture began with 7500 bacteria. Its growth can be modelled using the formula $N = 7500(2)^{\frac{t}{36}}$, where N is the number of bacteria after t hours.

(a) What is the doubling time?

The doubling time is 36 hours.

(b) How many bacteria are present after 36 hours?

$t = 36$

$$N = 7500(2)^{\frac{36}{36}}$$
$$= 15000$$

15000 bacteria are present.

(c) How many bacteria are present after 72 hours? How does this relate to the doubling time?

$t = 72$

$$N = 7500(2)^{\frac{72}{36}}$$
$$= 30000$$

$\frac{72}{36} = 2$

The bacteria are doubled twice.

Half-Life: Half-life is the time it takes for a quantity to decay to half its original amount.

The relation for doubling is $M = M_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where M represents the final quantity,

M_0 represents the initial quantity,
t represents time
h represents the half-life, and
the base "$\frac{1}{2}$" indicates half-life

2. All living organisms contain a known concentration of 1 part per trillion parts of carbon-14. Carbon-14 is a radioactive element. It is used to date ancient artefacts because it has a half-life of about 5730 years after the organism dies. The formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$ is used to calculate the concentration, C, in parts per trillion, remaining n years after death.

(a) What is the initial concentration of carbon-14 as given in the formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$?

1 is the initial concentration.

(b) What would be the concentration of carbon-14 in a piece of cloth (made from plant fibres) after 5730 years?

$$C = \left(\frac{1}{2}\right)^{\left(\frac{5730}{5730}\right)}$$

$$= \frac{1}{2}$$

n

↓

(c) What would be the concentration of carbon-14 in an animal bone after 50 000 years? Round your answer to five decimal places.

$$C = \left(\frac{1}{2}\right)^{\left(\frac{50000}{5730}\right)}$$

$$= 0.00236$$

3. *E. coli* is a very harmful type of bacteria that can be found in meat that is improperly stored or handled. The relation $N = N_0(2)^{\frac{t}{20}}$ estimates the number of *E. coli*, *N*, of an initial sample of N_0 bacteria after t minutes, at 37°C (body temperature), under optimal conditions.

(a) What is the doubling time of *E. coli*?

20 minutes

60 min

N_0

(b) If a sample of *E. coli* contains 5000 bacteria, how many will there be after 1 hour?

$$N = 5000(2)^{\left(\frac{60}{20}\right)}$$

$$= 5000(2)^3$$

$$= 40000 \text{ bacteria}$$

$1 \times 24 \times 60$

(c) If a sample of *E. coli* contains 1000 bacteria, how many will there be after 1 day?

N_0

$$N = 1000(2)^{\left(\frac{1440}{20}\right)}$$

4. The deer population of a national park was 250 deer 12 years ago. Today, there are 500 deer. Assuming the deer population has experienced exponential growth, write a relation representing the size of the deer population in the park. Use your relation to project the deer population in 25 years.

5. The relation $T = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$ can be used to determine the length of time, t, in hours, that milk of a certain fat content will remain fresh. T is the storage temperature, in degrees Celsius.

(a) What is the freshness half-life of milk?

10 hours..

(b) How long will milk keep fresh at 22°C? $t = ?$ $T = 22$

$22 = 190\left(\frac{1}{2}\right)^{\left(\frac{t}{10}\right)}$

$t = 20, \; 190\left(\frac{1}{2}\right)^{\left(\frac{20}{10}\right)} = 47.5$

$t = 31, \; 190\left(\frac{1}{2}\right)^{\left(\frac{31}{10}\right)} = 22.16 \rightarrow$ 31 hours

$t = 32, \; 190\left(\frac{1}{2}\right)^{\left(\frac{32}{10}\right)} = 20.68$ Answer statement

(c) How long will milk keep fresh at 4°C?

$t = 55, \; 190\left(\frac{1}{2}\right)^{\left(\frac{55}{10}\right)} = 4.19$

$t = 56, \; 190\left(\frac{1}{2}\right)^{\left(\frac{56}{10}\right)} = 3.91 \rightarrow$ 56 hours.

7. The remaining concentration of a particular drug in a person's bloodstream is modelled by the relation $C = C_0\left(\frac{1}{2}\right)^{\frac{t}{4}}$, where C is the remaining concentration of drug in the bloodstream in milligrams per millilitre of blood, C_0 is the initial concentration, and t is the time, in hours, that the drug is in the bloodstream.

a) What is the half-life of this drug?

b) A nurse gave a patient this drug. The concentration was 40 mg/mL, at 10:15 A.M. What will the concentration at

i) 3:15 P.M.? 5 hrs **ii)** 10:00 P.M.? 11.75 hrs.

c) A second dose of the drug needs to be given to this patient when the concentration of drug in the bloodstream is down to 0.5 mg/mL. Estimate after how many hours this would occur.

10. A general relation between speed and collision rates is "1 km faster results in a 3% increase in the collision rate." On a specific stretch of road, the collision rate is 0.534 collisions per million vehicle kilometres, when vehicles travel at an average speed of 80 km/h.

a) What is meant by "collisions per million vehicle kilometres"?

b) The relation that represents the collision rate for this stretch of road is $R = 0.534(1.03)^s$, where R is the collision rate, in collisions per million vehicle kilometres, and s is the average vehicle speed. What would be the collision rate if the average speed increases to

i) 90 km/h? **ii)** 120 km/h?