Worksheet 5-7: Explore Exponential Growth and Decay with Technology

(b) Use a graphing calculator to graph the relation. (Hint: Follow th

Press 2nd [STATPLOT]. Select **4:PlotsOff**. Press ENTER. (To cal amount, and x is number of changes over time, with an initial amount of a, and a growth factor of b when $b > 1$.

1. Animal Population

In a national park, a wolf population increased by a growth factor of 1.078 per year over a ten-year period, beginning in 1997. The formula $P = 124(1.078)^n$ modelled the wolf population after n years.

(a) Without graphing, state the wolf population in 1997. Explain how you get your answer.

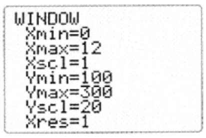
$P = 124(1.078)^n$ $\qquad\qquad$ $y = a(b)^x$

a is the initial amount.

Since a = 124, the wolf population in 1997 (when n = 0) was 124.

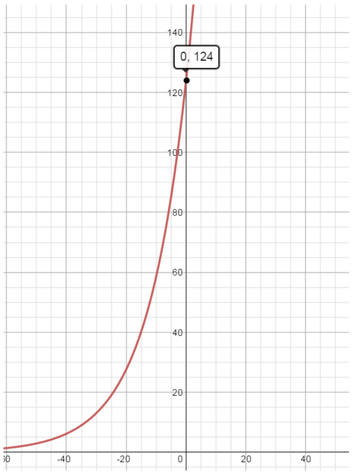
(b) Use a graphing calculator to graph the relation. (Hint: Follow the steps below.)

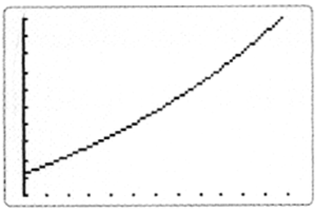
Press 2nd [STATPLOT]. Select **4:PlotsOff**. Press ENTER. (To clear scatter plots in RAM)
Press Y=. If necessary, clear all equations.
Type 124 × 1.078 ^, and then press X,T,θ,n. (Press X,T,θ,n for the variable x.)
Press WINDOW. Use the window settings shown.



Press GRAPH.

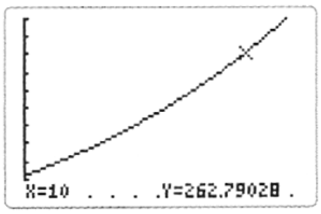
Use Desmos.com





(c) What was the wolf population in 2007?

Press 2nd [CALC]. Select **1:value**.
Press ENTER, then enter 10 for **X=**. (Year 2007 is 10 years after 1997, $x = 10$ represents 2007)
Press ENTER.



$n = 2007 - 1997 = 10$

Sub $n = 10$ into equation

$P = 124(1.078)^{10}$

$= 262.79$

$\doteq 263 \rightarrow$ Round to nearest unit.

$\therefore$ The wolf population was 263 in 2007.

Exponential Decay: $y = a(b)^x$, where y is the remaining amount, and x is number of changes over time, with an initial amount of a, and a decay factor of b when $0 > b > 1$.

2. Light Intensity

A sheet of translucent glass 1 mm thick reduces the intensity of the light passing through it. Light intensity is further reduced as more sheets of glass are placed together, as shown in the table.

x	**Number of Glass Sheets**	0	1	2	3	4	5	6	7	8
y	**Light Intensity (%)**	100	89.1	79.4	70.7	63.0	56.1	50.0	44.5	39.7

b = common ratio = 0.89
(divide consecutive terms) $\frac{89.1}{100} = 0.891$ 0.891 0.890 0.890 0.890 0.891 0.890 0.892

(a) What is the decay factor for the relation?

b = 0.89 The decay factor is 0.89.

(b) What is the initial amount of light intensity?

a = 100% The initial amount of light intensity is 100%.

(c) Write the formula that models the above situation.

$y = 100(0.89)^x$

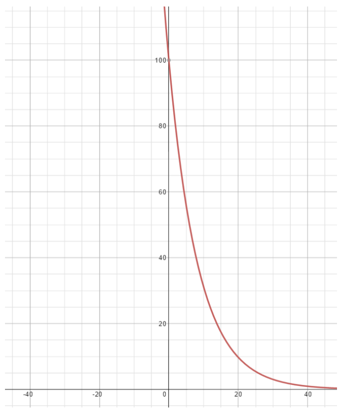
(d) The reduction rate of a sheet of glass is the percent by which the light intensity is reduced by adding a sheet of glass to a viewing panel. What is the light intensity reduction rate of a single sheet of glass? Express your answer as a percent.

1 - 0.89 = 0.11

The intensity reduction rate is 11%

(e) Use a graphing calculator to graph the relation. Refer to steps for Question #1 (b).
Press WINDOW. Use the window settings shown.

Use Desmos.com to graph.



(f) How many sheets of glass are needed to reduce the light intensity by one half?

y = 50 i.e. half of 100 a = 100 b = 0.89 x = ?

You need to find x, the exponent. How? By Trial-and-Error i.e. testing different x-values!

$50 = 100(0.89)^x$ $x = 5$, $y = 100(0.89)^5 = 55.84$ (not 50 yet!)

$x = 6$, $y = 100(0.89)^6 = 49.70$ (close, and this is it!)

6 sheets of glass are needed to reduce light intensity by half.

(g) How many sheets of glass are needed to reduce the light intensity to about 25%?
Press TRACE. Use the cursor keys to move the point shown on the graph until the value of Y is as close as possible to 25.

y = 25 a = 100 b = 0.89 x = ?

You need to find x, the exponent. How? By Trial-and-Error i.e. testing different x-values!

$25 = 100(0.89)^x$ $x = 11$, $y = 100(0.89)^{11} = 27.75$ (not 25 yet!)

$x = 12$, $y = 100(0.89)^{12} = 24.70$ (close, this is it!)

12 sheets of glass are needed to reduce light intensity to 25.

3. Cells in a culture are growing by a factor of 3.45 per day. The number of cells in the culture, N, can be estimated using the formula $N = 1000(3.45)^d$, where d is the number of days.

(a) Use a graphing calculator to plot the graph of this relation.

Press [2nd] [STATPLOT]. Select **4:PlotsOff**. Press [ENTER]. (To clear scatter plots in RAM)
Press [Y=]. If necessary, clear all equations.
Type **1000** [×] **3.45** [^], and then press [X,T,θ,n]. (Press [X,T,θ,n] for the variable x.)
Press [ZOOM]. Select "ZoomFit".

Go to Desmos.com to graph instead of using the graphing calculator.

(b) What is the growth factor of the cells in the culture?

b = 3.45 The growth factor is 3.45.

(c) How many cells does this culture begin with?

a = 1000 This culture begins with 1000 cells.

(d) How many cells would there be after 1 day?
Press [2nd] [CALC]. Select **1:value**.

Find N = ? when d = 1

$N = 1000(3.45)^1$
$= 3450$ There will be 3450 cells after 1 day.

(e) How many cells would there after 5 days?
Press [2nd] [CALC]. Select **1:value**.

Find N = ? when d = 5

$N = 1000(3.45)^5$
$= 488760$ There will be 488760 cells after 5 days.

4. A deer population is declining by 2.2% per year. The population can be modelled using the formula $P = 240(0.978)^n$, where P is the population after n years.

(a) Use a graphing calculator to plot the graph of this relation.

Press [2nd] [STATPLOT]. Select **4:PlotsOff**. Press [ENTER]. (To clear scatter plots in RAM)
Press [Y=]. If necessary, clear all equations.
Type in the equation. Press [X,T,θ,n] for x.
Press [ZOOM]. Select "ZoomFit".

Go to Desmos.com to graph instead of using the graphing calculator.

(b) What is the current deer population?

a = 240 The current deer population is 240.

(c) What is the declining rate of the deer population per year? Express your answer as a percent.

Declining rate = the percentage being taken away each year

Declining rate per year is 2.2%.

If declining rate is not given in the question,
declining rate = 1 - decay factor which is the base of the power
declining rate = 1 - 0.978 = 0.022 = 2.2%

(d) What is the decay factor?

b = 0.978 The decay factor is 0.978 or 97.8%.

(e) What will be the expected deer population after 8 years?
Press [2nd] [CALC]. Select **1:value**.

Find P = ? when n = 8

$P = 240(0.978)^8$
$= 201$ The expected deer population will be 201.

(f) How long does it take to reduce the deer population by one half?
P = 240/2 = 120 i.e. half of 240 Find x = ? when n = 120 By Trial-and-Error

$120 = 240(0.978)^n$

$n = 31$, $y = 240(0.978)^{31} = 120.42 > 120$ (close, but not 120 yet)

$n = 32$, $y = 240(0.978)^{32} = 117.78 < 120$ (pass 120, and this is it!)

It takes about 32 years to reduce the deer population by one half.