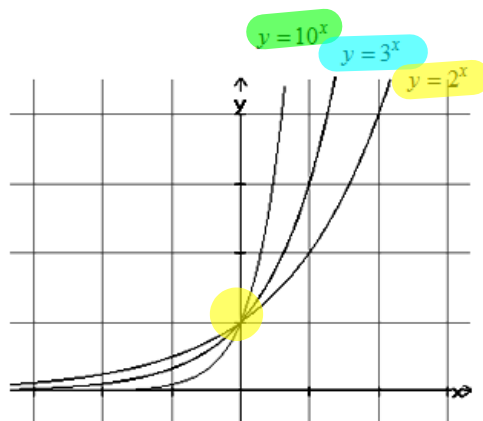


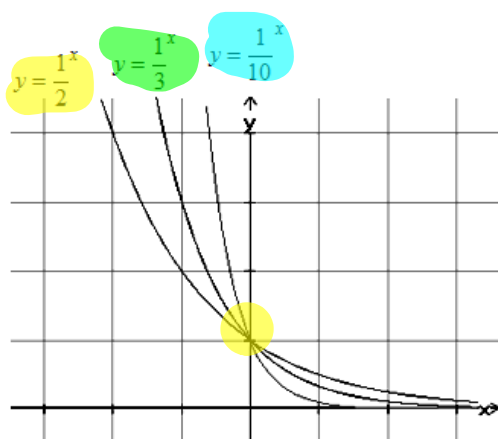
Worksheet 5-6: Exponential Relations: $y = b^x$ and $y = a(b)^x$

Properties of Exponential Relations $y = b^x$:

- A relation of the form $y = b^x$, where $b > 0$ and $b \neq 1$, is exponential.
- If $b > 1$, moving left to right, the graph increases very slowly for negative x -values and increases more rapidly for positive x -values. The graph is almost horizontal on the left and very steep on the right.



- If $0 < b < 1$, moving from left to right, the graph decreases very rapidly for negative x -values and decreases more slowly for positive x -values. The graph is almost horizontal on the right and very steep on the left.



- The y-intercept is 1.
- There is no x-intercept.
- The “growth” factor or “decay” factor is the base of the power, b , which is the common ratio between successive y -values.

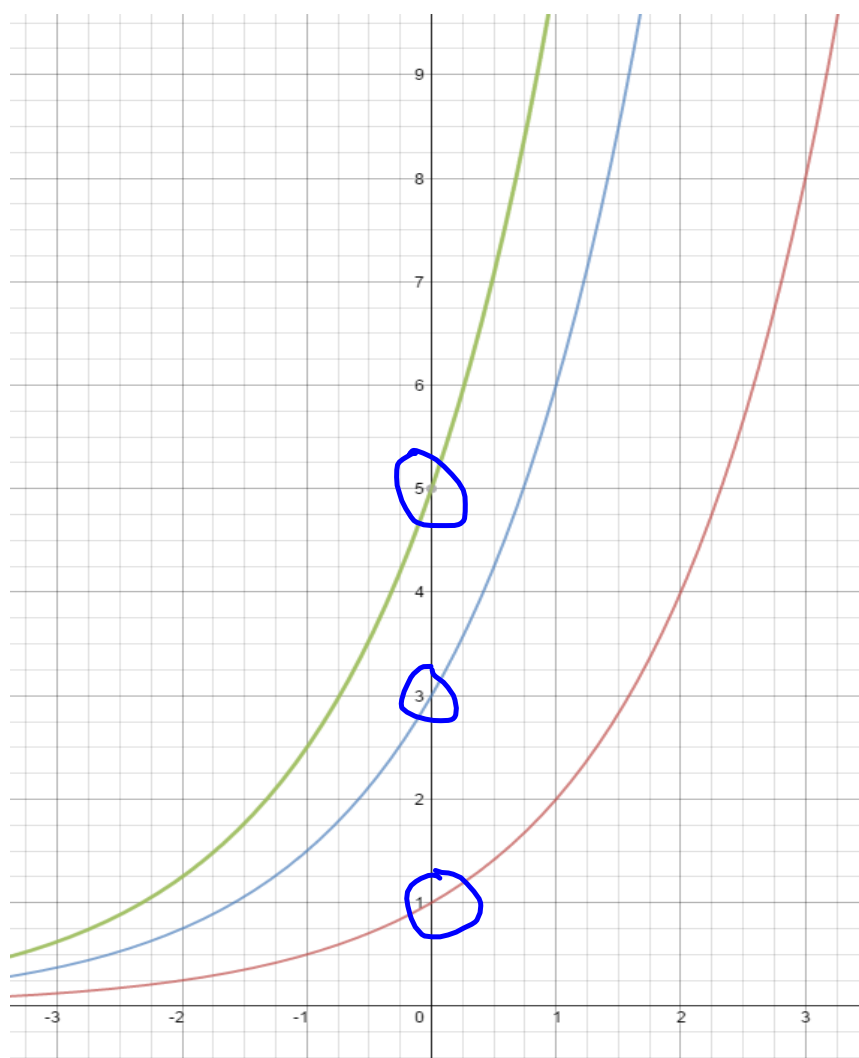
Investigation: Exponential Relations $y = a(b)^x$, where a is the initial amount or y-intercept

1. Graph $y = 2^x$, $y = 3(2^x)$, and $y = 5(2^x)$ on the same axes and compare.

x	$y = 2^x$ $a=1$
-3	
-2	
-1	
0	
1	
2	
3	

x	$y = 3(2^x)$ $a=3$
-3	
-2	
-1	
0	
1	
2	
3	

x	$y = 5(2^x)$ $a=5$
-3	
-2	
-1	
0	
1	
2	
3	



Conclusions: Compare the shape and y-intercept of the relations.

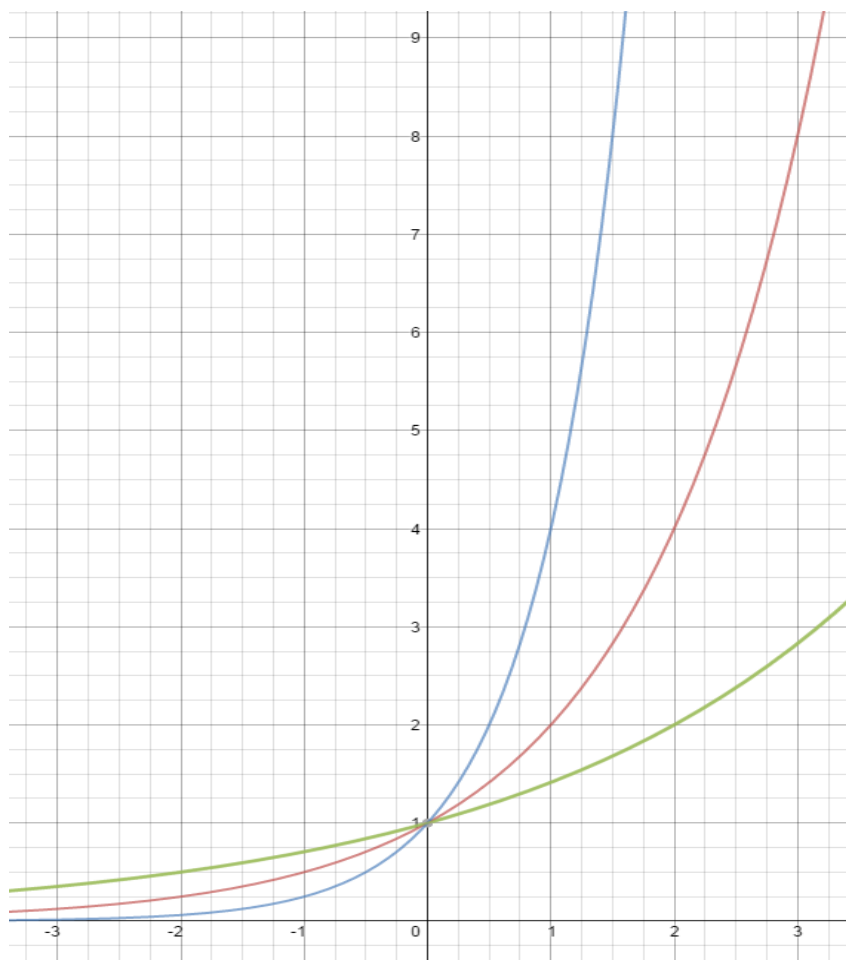
Investigation: Exponential Relations $y = b^x$, where x is multiplied or divided

2. Graph $y = 2^x$, $y = 2^{2x}$, and $y = 2^{\frac{x}{2}}$ on the same axes and compare.

x	$y = 2^x$
-3	
-2	
-1	
0	
1	
2	
3	

x	$y = 2^{2x}$
-3	
-2	
-1	
0	
1	
2	
3	

x	$y = 2^{\frac{x}{2}}$
-3	
-2	
-1	
0	
1	
2	
3	

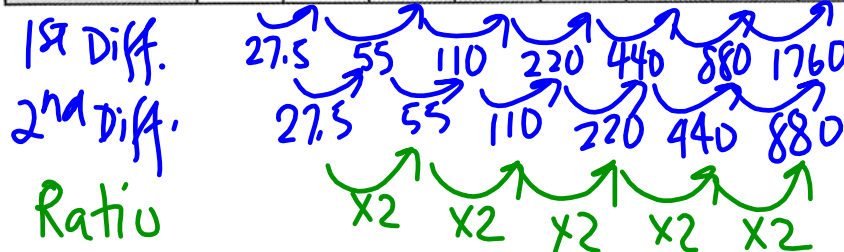


Conclusions: Compare the shape and y-intercept of the relations.

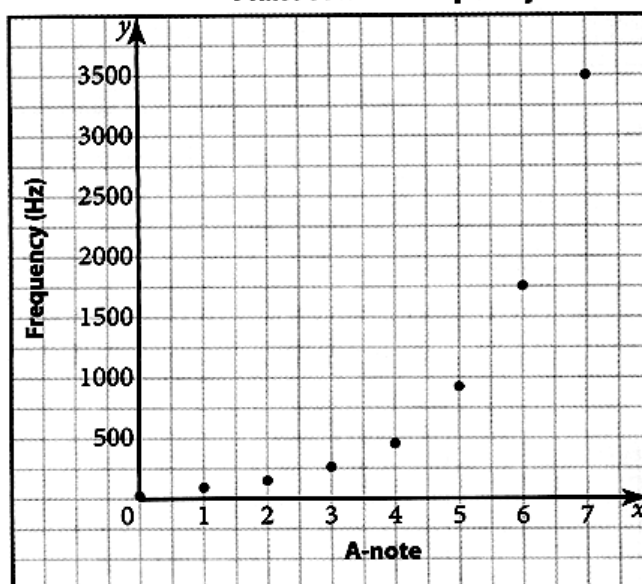
3. Musical Scale

Middle A on a piano is known as A4. Its sound wave has a frequency of 440 cycles per second, also written as 440 Hertz (Hz). The table and the graph show the frequencies of each of the eight A-notes on a piano.

A-note	0	1	2	3	4	5	6	7
Frequency (Hz)	27.5	55	110	220	440	880	1760	3520



Piano A-note Frequency



(a) Describe the graph.

The graph is in the shape of a curve that goes up to the right. It starts at 0.

(b) Does the relationship between the A-notes on a piano and their frequencies model an exponential growth?

Though the curve looks like half of a parabola, the first and second differences are not constant, so the relationship cannot be quadratic. Since the ratios among consecutive terms are constant, the relationship is exponential. The relationship between the A-notes and their frequencies is an exponential growth with a constant ratio of 2.

6. York Region's population, P , is projected to grow until 2031 based on the relation $P = 610\,000(1.029)^n$, where n is the number of years after 1996.
 1997-1996 = 1 2015-1996 = 19 = n

a) Sketch a graph of this relation.

b) What is the P -intercept? What does it represent?
 610000 → Population in 1996

c) What is the projected population of York Region in

i) 2015? →

ii) 2031?

$$n = 2031 - 1996 = 35$$

$$610000(1.029)^{19} = 1050079 \text{ (to "one")}$$

$$610000(1.029)^{35}$$

$$= 1664735$$

$a = \text{initial amount}$

7. A pressure reader is used to measure the sound intensity of a bell.
 The relation $P = 200(0.5)^t$ estimates the sound pressure, P , in pascals, after t seconds.

Dependent
time \uparrow *independent*

a) Sketch a graph of this relation.

b) What is the P -intercept? What does it represent?

Sound pressure in Pascal when $t=0$ or initially when start to measure.

c) What was the sound pressure after

i) 1 s?

ii) 2 s?

$$\begin{aligned} \text{(i) } P &= 200(0.5)^t \\ &= 200(0.5)^1 \\ &= 100 \end{aligned}$$

$$\begin{aligned} \text{(ii) } P &= 200(0.5)^2 \\ &= 50 \end{aligned}$$