Worksheet 5-5: Rates of Change/Growth

How do we identify linear, quadratic or exponential relations?

1. **Classify by Equation:**
   - ☺ Linear: x is a first-degree variable (Exponent is 1).
   - ☺ Quadratic: x is a second-degree variable (Exponent is 2).
   - ☺ Exponential: x itself is the exponent.

   $y=1$ is still linear
2. **Classify by Graph:**
   - ☺ Linear: the graph is a straight line.
   - ☺ Quadratic: the graph is a parabola (U-shape).
   - ☺ Exponential: the graph is an exponential curve (J-shaped).

3. **Classify by Finite Differences:**
   - ☺ Linear: first differences are constant.
   - ☺ Quadratic: first differences increase by a constant value (adding). Second differences are constant.
   - ☺ Exponential: first differences increase by a constant factor (multiplying), a common ratio.

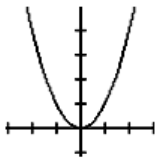
Check for Understanding:

1. Without graphing, classify each of the following as linear, quadratic, or exponential growth.

(a) $3x-4y=12$	Linear	Quadratic	Exponential
(b) $y=2x^2+3$	Linear	Quadratic	Exponential
(c) $y=\left(\frac{1}{5}\right)^x$	Linear	Quadratic	Exponential
(d) $y=3(1.05)^x$	Linear	Quadratic	Exponential

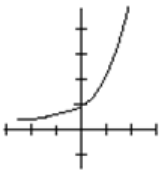
2. Examine each graph and classify as linear, quadratic, or exponential growth.

(a)



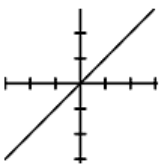
Linear Quadratic Exponential

(b)



Linear Quadratic Exponential

(c)



Linear Quadratic Exponential

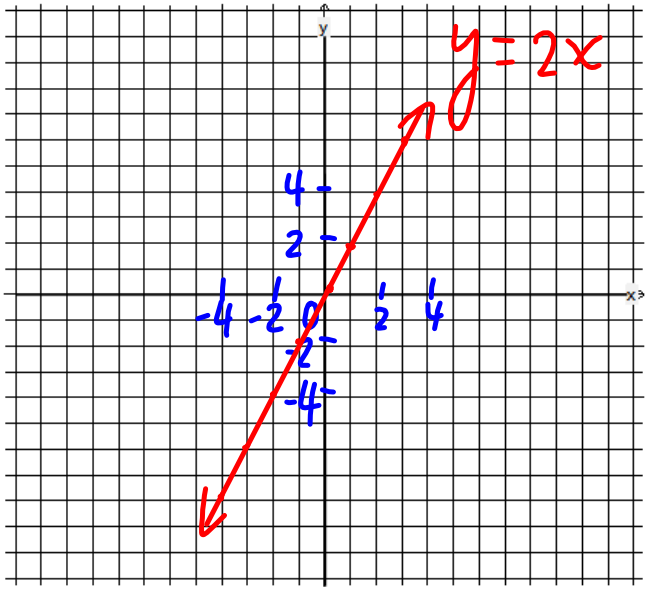
Investigation:

1. Graph $y = 2x$.

Linear

x	$y = 2x$	First Difference
-3	$2(-3) = -6$	
-2	$2(-2) = -4$	$-4-(-6) = 2$
-1	$2(-1) = -2$	$-2-(-4) = 2$
0	$2(0) = 0$	$0-(-2) = 2$
1	$2(1) = 2$	$2-0 = 2$
2	$2(2) = 4$	$4-2 = 2$
3	$2(3) = 6$	$6-4 = 2$

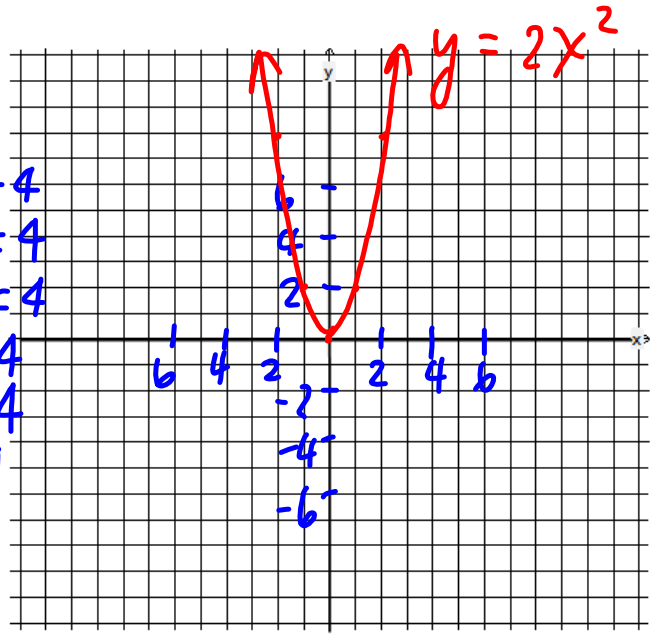
slope



2. Graph $y = 2x^2$.

x	$y = 2x^2$	First Difference	2nd Diff
-3	$2(-3)^2 = 18$		
-2	$2(-2)^2 = 8$	$8-18 = -10$	$-6-(-10) = 4$
-1	$2(-1)^2 = 2$	$2-8 = -6$	$-2-(-6) = 4$
0	$2(0)^2 = 0$	$0-2 = -2$	$2-(-2) = 4$
1	$2(1)^2 = 2$	$2-0 = 2$	$6-2 = 4$
2	$2(2)^2 = 8$	$8-2 = 6$	$10-6 = 4$
3	$2(3)^2 = 18$	$18-8 = 10$	

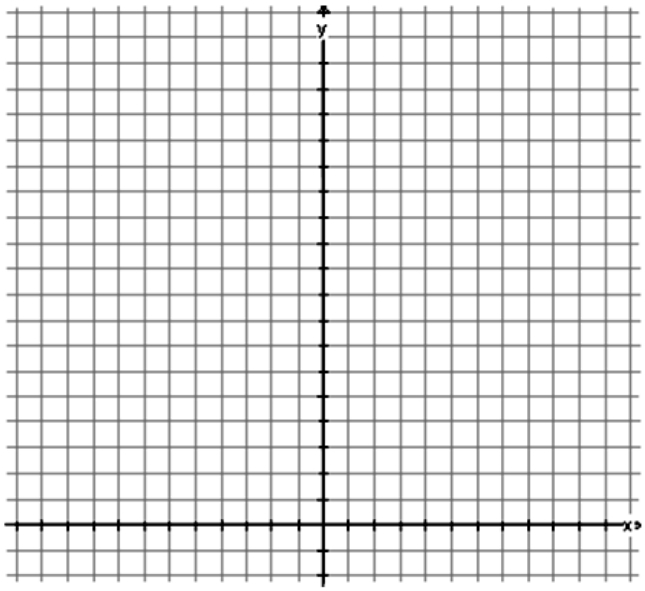
2nd differences are constant



3. Graph $y = 2^x$.

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

x	$y = 2^x$	~~First Difference~~ Ratio
-3	$2^{-3} = \frac{1}{8}$	
-2	$2^{-2} = \frac{1}{4}$	$\frac{1}{4} \div \frac{1}{8} = 2$
-1	$2^{-1} = \frac{1}{2}$	$\frac{1}{2} \div \frac{1}{4} = 2$
0	$2^0 = 1$	$1 \div \frac{1}{2} = 2$
1	$2^1 = 2$	$2 \div 1 = 2$
2	$2^2 = 4$	$4 \div 2 = 2$
3	$2^3 = 8$	$8 \div 4 = 2$



$\frac{1}{4} \div \frac{1}{8}$

$= 1 \div 4 \div (1 \div 8)$

4. Graph $y = \left(\frac{1}{2}\right)^x$.

$\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = 2^3$

x	$y = \left(\frac{1}{2}\right)^x$	~~First Difference~~ Ratio
-3	$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$	
-2	$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$	
-1	$\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$	
0	$\left(\frac{1}{2}\right)^0 = 1$	
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$	
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$	
3	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$