Worksheet 5-4: Investigating Exponential Relationships

1. Paper-folding Investigation: Exponential Growth *(Increasing)*

(a) Take a large rectangular sheet of paper and fold it in half. Unfold it and count the number of rectangles formed by the crease. Record the number of folds and the number of rectangles in the table provided in part (c), and then refold the paper.

(b) Fold the paper in half again. Unfold and record the number of rectangles formed by the creases. Refold the paper again.

(c) Continue folding in half and recording the number of rectangles until you can no longer fold the paper. Record your findings in the following table.

x y

Number of Folds	Number of Rectangles	
0	1	$\times 2$
1	2	$\times 2$
2	4	$\times 2$
3	8	$\times 2$
4	16	$\times 2$
5	32	$\times 2$
6	64	$\times 2$
7	128	$\times 2$
8	256	

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$

(d) Graph the relation, showing the number of folds on the horizontal axis and the number of rectangles on the vertical axis. (Graph on next page.)

(e) If x represents the number of folds and y represents the number of rectangles, write an equation for the relation.

$$y = 2^x$$

For exponential growth,

$y = b^x$ **, where** $b > 1$.

b is the base.

2. Paper-cutting Investigation: Exponential Decay *(Decreasing)*

(a) Take a large rectangular sheet of paper and cut it into halves. Put away the half you cut out. Record the number of cuts and the remaining portion of the original paper after the cut, as a fraction, in the table provided in part (c).

(b) Cut the paper into halves again. Put away the half you cut out and record the remaining portion as a fraction of the original paper.

(c) Continue cutting the remaining paper into halves and recording the number of cuts and the remaining portion of the original paper until you can no longer cut the paper. Record your findings in the following table.

Number of Cuts	Remaining Portion of Original Paper
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$
5	$\frac{1}{32}$
6	$\frac{1}{64}$
7	$\frac{1}{128}$
8	$\frac{1}{256}$

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

(d) Graph the relation, showing the number of cuts on the horizontal axis and the remaining portion as a fraction of the original paper on the vertical axis. (Graph on next page.)

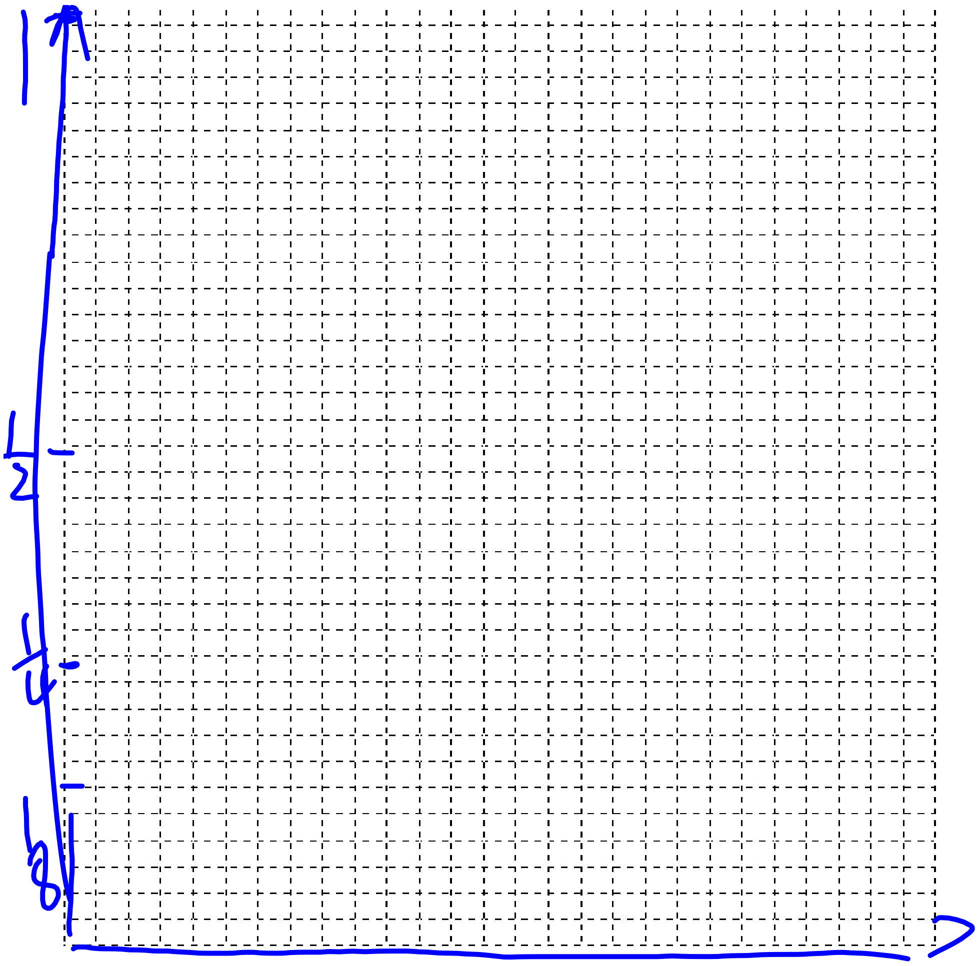
Hint: you may want to set a bigger scale for your vertical axis.
For example: use 20 units as 1 to graph "fractions".

(e) If x represents the number of cuts and y represents the remaining portion of the original paper, write an equation for the relation.

$$y = \left(\frac{1}{2}\right)^x$$

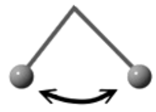
For exponential growth,

$y = b^x$, where $0 < b < 1$.


1
1/2
1/4
1/8

3. Pendulum Motion

A large pendulum was set in motion. With each complete swing, the pendulum's maximum distance from its rest position decreased. A motion sensor was used to obtain the data after every 5 swings, and a scatter plot of the data is given below.

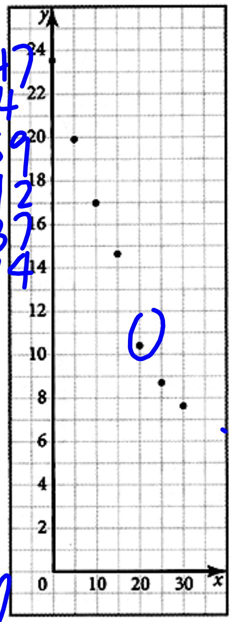


Pendulum Motion

Number of Swings	Maximum Distance (cm)
0	23.5
5	19.9
10	17.0
15	14.6
20	10.4
25	8.7
30	7.6

Ratio

$19.9 \div 23.5 = 0.847$
$17 \div 19.9 = 0.854$
$14.6 \div 17 = 0.859$
$10.4 \div 14.6 = 0.712$
$8.7 \div 10.4 = 0.837$
$7.6 \div 8.7 = 0.874$



(a) Describe the graph.

- the dots follow a curve that goes down to the right.
- The graph starts at an initial distance of 23.5 cm.
- It is the graph of an exponential decay.

✓

(b) Calculate the ratio between successive distances. Is the relationship between the number of swings and the maximum distance of the pendulum swing exponential? Explain.

The ratio between successive distances is approximately 0.85.
Yes, it's exponential because the rate (ratio) of change is quite constant.

(c) Is this relationship an example of *exponential growth* or *exponential decay*?

This is an example exponential decay.