

Worksheet 5-2: Zero and Negative Exponents

Investigation: Zero and Negative Exponents

Quotient	Expanded Form	Number of Factors	Single Power	Evaluated
$\frac{5^4}{5^4}$	$\frac{5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 1$	0	5^0	1
$\frac{(-3)^5}{(-3)^5}$	$\frac{(-3)(-3)(-3)(-3)(-3)}{(-3)(-3)(-3)(-3)(-3)} = 1$	0	$(-3)^0$	1
$\frac{2^3}{2^3}$	$\frac{2 \times 2 \times 2}{2 \times 2 \times 2}$	0	2^0	1
$\frac{4^2}{4^3}$	$\frac{4 \times 4}{4 \times 4 \times 4} = \frac{1}{4}$	-1	4^{-1}	$\frac{1}{4} = \frac{1}{4}$
$\frac{2^3}{2^5}$	$\frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$	-2	2^{-2}	$\frac{1}{2^2} = \frac{1}{4}$
$\frac{(-10)^3}{(-10)^6}$	$\frac{(-10)(-10)(-10)}{(-10)(-10)(-10)(-10)(-10)(-10)}$	-3	$(-10)^{-3}$	$\frac{1}{(-10)^3} = -\frac{1}{1000}$

Zero Exponent: $x^0 = 1$

Negative Exponent: $x^{-n} = \frac{1}{x^n}$

$\frac{1}{x^{-n}} = x^n$

$$\frac{1}{x^{-n}} = \frac{1}{\frac{1}{x^n}} = x^n$$

Practice:

1. Write each as a single power with positive exponents, then evaluate as integers or fractions.

(a) 8^{-2}

$= \frac{1}{8^2}$ ✓

$= \frac{1}{64}$ ✓

(b) 100^0

$= 1$

(c) $(-4)^{-1}$

$= \frac{1}{(-4)^1}$ ✓

$= -\frac{1}{4}$ ✓
 ↑
 must be outside

(d) $\frac{1}{3^{-2}}$

$= 3^2$ ✓
 $= 9$ ✓

$= \frac{1}{\frac{1}{3^2}}$
 $= 3^2 = 9$

2. Use exponent rules to simplify each expression then evaluate as integers or fractions.

$$\begin{aligned}
 \text{(a)} \quad & 4^3 \times 4^{-5} \\
 & = 4^{3+(-5)} \\
 & = 4^{-2} \\
 & = \frac{1}{4^2} \\
 & = \frac{1}{16}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{(-2)^2}{(-2)^{-5}} \\
 & = (-2)^{2-(-5)} \\
 & = (-2)^7 \\
 & = -128
 \end{aligned}$$

(c) $\left(\frac{4^2}{4^5}\right)^2$ Do what's inside brackets first!

$$\begin{aligned}
 & = \left(4^{2-5}\right)^2 \\
 & = \left(4^{-3}\right)^2 \\
 & = 4^{-6} \\
 & = \frac{1}{4^6} \\
 & = \frac{1}{4096}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \left(\frac{5^3 \times 5^4}{5^7}\right)^{105} \\
 & = \left(\frac{5^{3+4}}{5^7}\right)^{105} \\
 & = \left(\frac{5^7}{5^7}\right)^{105} \\
 & = (5^{7-7})^{105}
 \end{aligned}$$

Try not to change to 1^{105}

$$\begin{aligned}
 & = (5^0)^{105} \\
 & = 5^0 \\
 & = 1
 \end{aligned}$$

Do Power of Power first!

Simplify with the rules until you can't simplify further!

3. Evaluate the power in each statement. Express your answers as integers or fractions.

(a) One kilobyte is 2^{10} bytes.

$$= 1024$$

(b) One byte is 2^{-10} kilobytes.

$$= \frac{1}{2^{10}}$$
$$= \frac{1}{1024}$$

(c) One megabyte is $(2^{10})^2$ bytes.

(d) One byte is $(2^{-10})^3$ gigabytes.