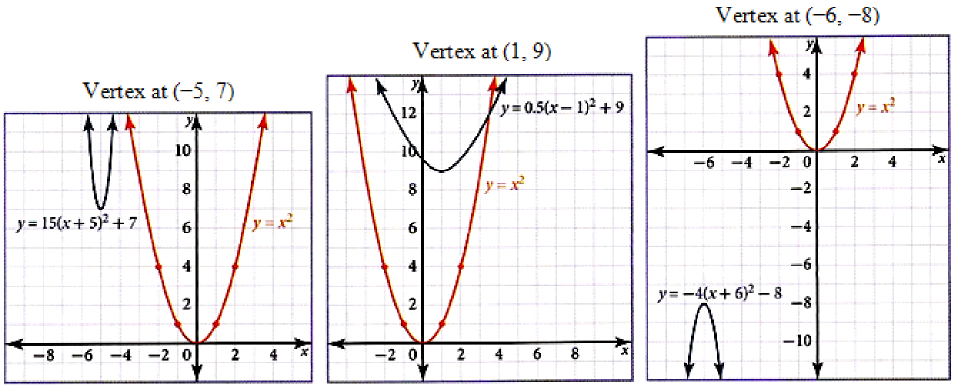
Read the front and complete the back in 5 minutes!

Summary of Transformations in Quadratic Relations

For any quadratic relation of the form $y = a(x-h)^2 + k$:



- The value of a determines the orientation (**upward or downward**) and shape (**stretch or compression**) of the parabola relative to the graph of $y = x^2$.

→ If $a > 0$ (a is greater than 0), the parabola opens upward.
→ If $a < 0$ (a is less than 0), the parabola opens downward, and is a reflection of $y = x^2$ in the x-axis.
→ If $-1 < a < 1$ (a is between −1 and 1), the parabola is vertically compressed relative to the graph of $y = x^2$.
→ If $a > 1$ or $a < -1$ (a is greater than 1 or less than −1), the parabola is vertically stretched relative to the graph of $y = x^2$.

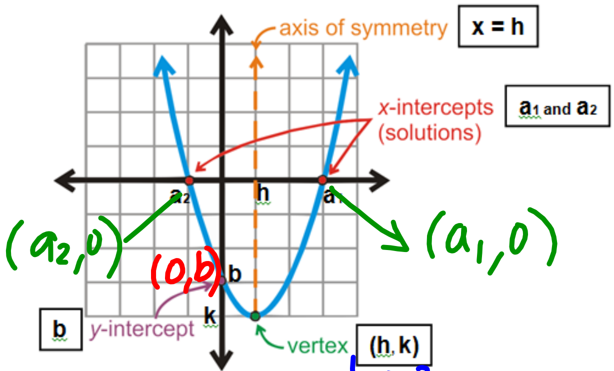
- The value of k determines the **vertical position** of the parabola.

→ If $k > 0$ (k is greater than 0), the parabola is vertically translated upward by k units relative to the graph of $y = x^2$, and the vertex of the parabola is k units above the x-axis.
→ If $k < 0$ (k is less than 0), the parabola is vertically translated downward by k units relative to the graph of $y = x^2$, and the vertex of the parabola is k units below x-axis.

- The value of h determines the **horizontal position** of the parabola.

→ If $h > 0$ (h is greater than 0), the vertex of the parabola is horizontally translated to the right of the y-axis by h units, relative to the graph of $y = x^2$.
→ If $h < 0$ (h is less than 0), the vertex of the parabola is horizontally translated to the left of the y-axis by h units, relative to the graph of $y = x^2$.

- The coordinates of the **vertex** of the parabola are **(h, k)**.



$h = 2$

For the quadratic function $f(x) = -3(x-2)^2 + 9$: $y = -\frac{1}{3}(x-2)^2 + 9$

(a) describe in words the transformation relative to the graph of $f(x) = x^2$.

−ve a h k compressed

The graph is reflected in the x-axis, stretched vertically by a factor of 3, $\frac{1}{3}$ shifted horizontally to the right by 2 units, and shifted vertically up by 9 units.

(b) write the coordinates of the vertex.

$v = (2, 9)$

(c) write the equation of the axis of symmetry.

$x = 2$

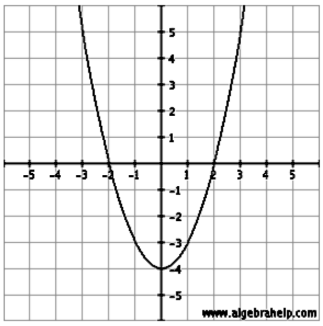
Worksheet 4-8: Interpret Graphs of Quadratic Relations

Besides the vertex, minimum or maximum y-value, and the axis of symmetry, x- and y-intercepts of a quadratic relation are also important information when interpreting a quadratic relation.

x-intercept is the x-coordinate of the point where the parabola crosses or touches the x-axis.
y-intercept is the y-coordinate of the point where the parabola crosses or touches the y-axis.

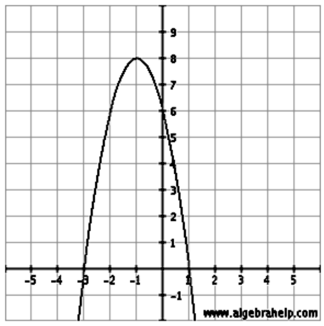
1. State the x- and y-intercepts of each quadratic relation.

(a)



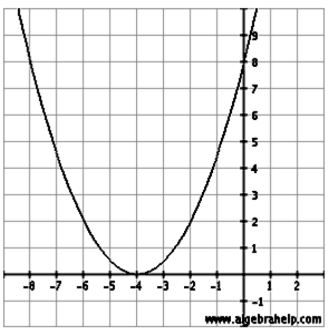
x-intercepts are -2 and 2.
y-intercept is -4.

(b)



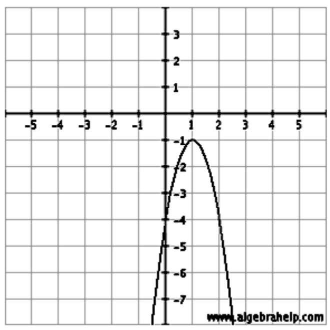
x-intercepts are -3 & 1.
y-intercept is 6

(c)



x-intercept is -4.
y-intercept is 8.

(d)



There is no x-intercept.
y-intercept is -4.

To find the x- and y-intercepts of a quadratic relation algebraically,

→ substitute $x = 0$ into the quadratic equation to find the y-intercept.

→ substitute $y = 0$ into the quadratic equation to find the x-intercept(s).

2. Find the y-intercept of each relation. Sub $x=0$

(a) $y = -3(x+2)^2 - 9$

$y = -3(0+2)^2 - 9$

$= -21$

y-intercept $= -21$

(b) $y = 0.1x^2 + 0.4x + 1.8$

$y = 0.1(0)^2 + 0.4(0) + 1.8$

$y = 1.8$

y-intercept $= 1.8$

(c) $y = 2(x-3)^2 + 12$

$y = 2(0-3)^2 + 12$

$= 2(-3)^2 + 12$

$= 2(9) + 12$

$= 18 + 12$

$= 30$

y-intercept $= 30$

(d) $y = -4x^2 - 8x - 9$

$y = -4(0)^2 - 8(0) - 9$

$= -9$

y-intercept $= -9$

3. Find the x-intercept of each relation.

(a) $y = 2(x-3)^2 - 8$ Vertex $= (3, -8)$

$0 = 2(x-3)^2 - 8$

$\frac{8}{2} = \frac{\cancel{2}(x-3)^2}{\cancel{2}}$

$4 = (x-3)^2$

$\pm\sqrt{4} = x - 3$

$2 = x - \cancel{3}$ or $-2 = x - \cancel{3}$

$+3 \quad +\cancel{3} \qquad +3 \quad +\cancel{3}$

$5 = x$ or $1 = x$

(b) $y = x^2 + x - 42$

$0 = x^2 + x - 42$

$0 = (x+7)(x-6)$

x^2	-42
x	7
x	-6
	$+1$

$x+7=0$ or $x-6=0$

$x = -7$ or $x = 6$

(c) $y = -3(x+5)^2 + 27$

$0 = -3(x+5)^2 + 27$

$-27 \qquad -27$

$\frac{-27}{-3} = \frac{-3(x+5)^2}{-3}$

$9 = (x+5)^2$

$\pm\sqrt{9} = x + 5$

$3 = x + 5$ or $-3 = x + 5$

$-5 \quad -5 \qquad -5 \quad -5$

$-2 = x$ or $-8 = x$

(d) $y = 2x^2 - 6x - 36$ GCF $= 2$

$0 = 2x^2 - 6x - 36$

$0 = 2(x^2 - 3x - 18)$

$0 = (x+3)(x-6)$

x^2	-18
x	3
x	-6
	-3

$x+3=0$ or $x-6=0$

$-3 \quad -3 \qquad +6 \quad +6$

$x = -3$ or $x = 6$

4. A construction worker drops his wrench. Its fall is modelled by the relation $h = -4.9t^2 + 342$, where h is the height above the ground, in metres, and t is the time after the wrench was dropped, in seconds. You must given h to find t or given t to find h.

(a) How far above the ground was the wrench when it was dropped?

h=? t=0

Find h when $t=0 \rightarrow$ find the y-intercept.

$h = -4.9t^2 + 342$

$= -4.9(0)^2 + 342$

$= 0 + 342$

$h = 342$

∴ The wrench was 342 m above when it was dropped.

(b) How far has the wrench fallen after 5 seconds?

h=? t=5

Find h when t=5,

$h = -4.9(5)^2 + 342$

$= -4.9(25) + 342$

$= 219.5$

But 219.5 m is where it was, how far had it been dropped is the distance from t=0 to t=5 ⇒

Distance fallen $= 342 - 219.5$

$= 122.5$

∴ The wrench has fallen 122.5m after 5 seconds.

(c) When will the wrench hit the ground?

t=? h=0

Find t when $h=0 \rightarrow$ x-intercept (reject negative h value)

$y = -4.9t^2 + 342$

$0 = -4.9t^2 + 342$

$+4.9t^2 \quad +4.9t^2$

$\frac{4.9t^2}{4.9} = \frac{342}{4.9}$

$t^2 = \frac{342}{4.9}$

$t = \pm\sqrt{\frac{342}{4.9}}$

$= 8.35$ or -8.35 (reject)

∴ The wrench will hit the ground after 8.35 s.

y-intercept

5. A football player kicks a football held 0.5 m above the ground. The football reaches a maximum height of 30 m at a horizontal distance of 18 m from the player.

k h

(a) Determine a quadratic relation that models the path of the football.

Vertex$(18, 30)$ y-intercept at $(0, 0.5)$

$a = ?$ → Sub into $y = a(x-h)^2 + k$

$x = 0$, $y = 0.5$, $h = 18$, $k = 30$

$0.5 = a(0-18)^2 + 30$

$0.5 = a(-18)^2 + 30$

$0.5 = 324a + 30$

$-30 \qquad -30$

$\frac{-30.5}{324} = \frac{324a}{324}$

$-0.09 = a$

∴ The quadratic relation is

$y = -0.09(x-18)^2 + 30$.

(b) At what horizontal distance from the player does the football hit the ground?

$x = ?$ $h = 0$

Find x when $h = 0$ → x-intercept

$y = -0.09(x-18)^2 + 30$

$0 = -0.09(x-18)^2 + 30$

$-30 \qquad -30$

$\frac{-30}{-0.09} = \frac{-0.09(x-18)^2}{-0.09}$

$\frac{30}{0.09} = (x-18)^2$

$\pm\sqrt{\frac{30}{0.09}} = x - 18$

$18.26 = x - 18$ or $-18.26 = x - 18$

$+18 \qquad +18$ $\qquad$ $+18 \qquad +18$

$36.26 = x$ $\qquad$ $-0.26 = x$ (reject)

∴ The horizontal distance is 36.26 m when the football hits the ground.