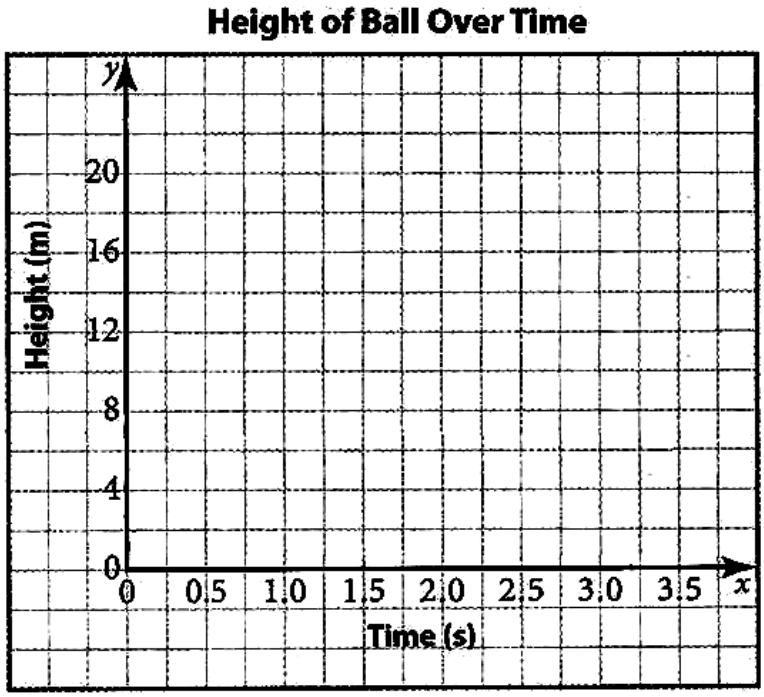
Real-Life Modelling of Quadratic Relations

The table shows a soccer ball's height above the ground over time after it was kicked in the air.

Time (s)	Height (m)
0	0.10
0.5	7.80
1.0	12.00
1.5	13.80
2.0	13.00
2.5	9.75
3.0	4.00

(a) Graph the data. Draw a smooth curve through the points.



(b) Why is "Time" graphed on the horizontal axis?

Time is the independent variable which is graphed on the horizontal axis. Therefore, Time is graphed on the horizontal axis.

(c) Describe the shape of the graph. State the direction of its opening.

The shape of the graph is a parabola, and its direction of opening is downward.

(d) What was the ball's maximum height?

The ball's maximum height is 13.8 m from the table, or 14 m from the given graph.

(e) For about how many seconds was the ball in the air?

The ball was in the air for about 3.2 seconds.

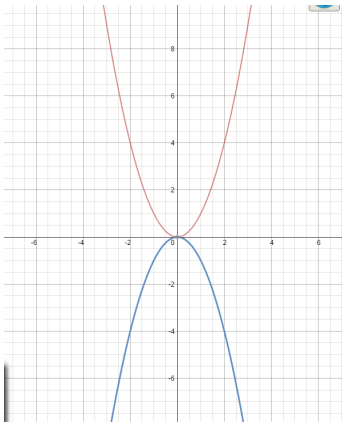
Worksheet 4-3: Quadratic Relations $y = ax^2$

Investigation 1: $y = ax^2$, "positive a" vs. "negative a"

On the same axes, graph $y = x^2$ and $y = -x^2$.

x	$x^2 = y$	(x, y)
2		
1		
0		
−1		
−2		

x	$-x^2 = y$	(x, y)
2		
1		
0		
−1		
−2		



Compare to the basic parabola $y = x^2$,

(a) how is $y = -x^2$ similar to $y = x^2$?

- same vertex
- same axis of symmetry
- same shape

(b) how is $y = -x^2$ different from $y = x^2$?

- "opposite" direction of opening

→ $y = x^2$ opens up
$y = -x^2$ opens down

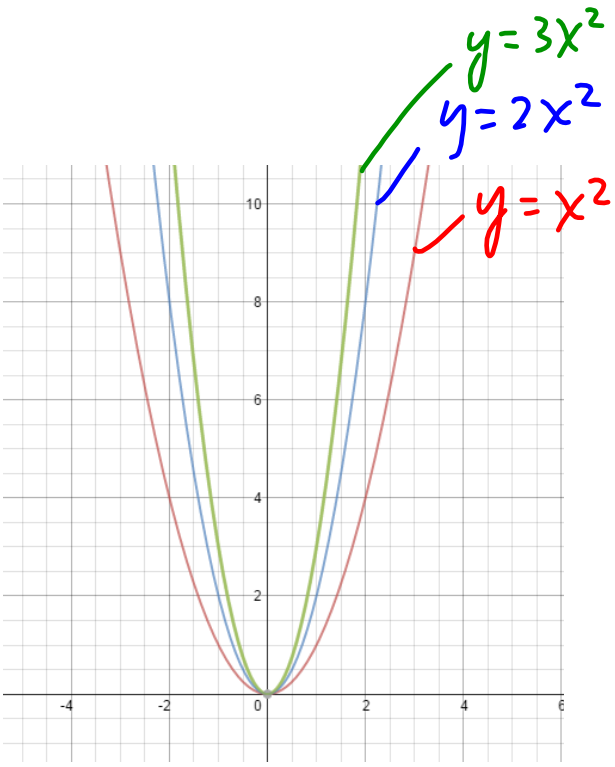
- $y = x^2$ has a minimum value of 0 but $y = -x^2$ has a maximum value of 0.

Investigation 2: $y = ax^2$, when a is positive and greater than 1

On the same axes, graph $y = x^2$, $y = 2x^2$ and $y = 3x^2$.

x	$2x^2 = y$	(x, y)
2	$2(2)^2 = 2(4) = 8$	(2, 8)
1	$2(1)^2 = 2(1) = 2$	(1, 2)
0	$2(0)^2 = 2(0) = 0$	(0, 0)
−1	$2(-1)^2 = 2(1) = 2$	(−1, 2)
−2	$2(-2)^2 = 2(4) = 8$	(−2, 8)

x	$3x^2 = y$	(x, y)
2	$3(2)^2 = 3(4) = 12$	(2, 12)
1	$3(1)^2 = 3(1) = 3$	(1, 3)
0	$3(0)^2 = 3(0) = 0$	(0, 0)
−1	$3(-1)^2 = 3(1) = 3$	(−1, 3)
−2	$3(-2)^2 = 3(4) = 12$	(−2, 12)



x	x^2
0	0
1	1
2	4
3	9

x	$y = x^2$	$y = 2x^2$	$y = 3x^2$	$y = 5x^2$
2	4	8	12	20
1	1	2	3	5
0	0	0	0	0
-1	1	2	3	5
-2	4	8	12	20

Challenge:

$y = -4x^2$

−16

−4

0

−4

−16

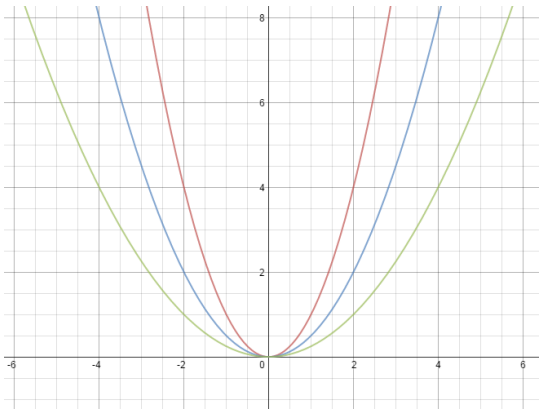
Similar	Different
- same vertex - same opening direction - same axis of symmetry - same minimum value of 0	- Different Shape

Investigation 3: $y = ax^2$, **when** a **is positive and less than 1**

On the same axes, graph $y = x^2$, $y = \frac{1}{2}x^2$ **and** $y = \frac{1}{4}x^2$. ****Hint: Use 2 units as 1 for the** y**-axis**

x	$\frac{1}{2}x^2 = y$	(x, y)
2	$(\frac{1}{2})(2)^2 = 2$	$(2, 2)$
1	$(\frac{1}{2})(1)^2 = \frac{1}{2}$	$(1, \frac{1}{2})$
0	$(\frac{1}{2})(0)^2 = 0$	$(0, 0)$
−1	$(\frac{1}{2})(-1)^2 = \frac{1}{2}$	$(-1, \frac{1}{2})$
−2	$(\frac{1}{2})(-2)^2 = 2$	$(-2, 2)$

x	$\frac{1}{4}x^2 = y$	(x, y)
2	$(\frac{1}{4})(2)^2 = 1$	$(2, 1)$
1	$(\frac{1}{4})(1)^2 = \frac{1}{4}$	$(1, \frac{1}{4})$
0	$(\frac{1}{4})(0)^2 = 0$	$(0, 0)$
−1	$(\frac{1}{4})(-1)^2 = \frac{1}{4}$	$(-1, \frac{1}{4})$
−2	$(\frac{1}{4})(-2)^2 = 1$	$(-2, 1)$



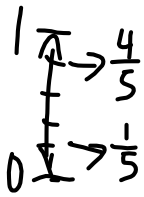
Similar

- same vertex
- same opening direction
- same axis of symmetry
- same minimum value of 1

Different

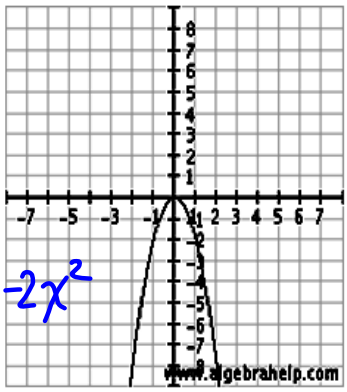
- different shape

x	$y = x^2$	$y = \frac{1}{2}x^2$	$y = \frac{1}{4}x^2$	$y = \frac{1}{5}x^2$
2	4	2	1	$\frac{4}{5}$
1	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{5}$
0	0	0	0	0
-1	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{5}$
-2	4	2	1	$\frac{4}{5}$



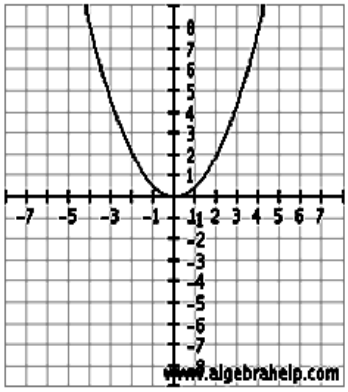
1. **Match the following graphs to their corresponding equations.**

$y = \frac{1}{2}x^2$ $y = -2x^2$ $y = 4x^2$ $y = -\frac{1}{3}x^2$

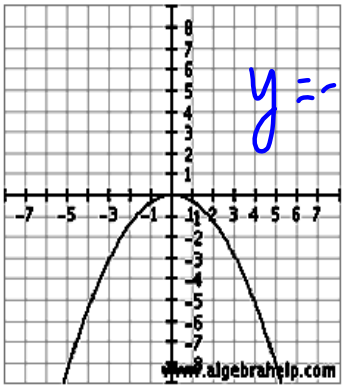


$y = -2x^2$

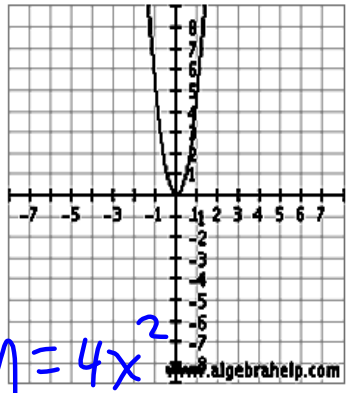
x	x^2
0	0
1	1
2	4



$y = \frac{1}{2}x^2$



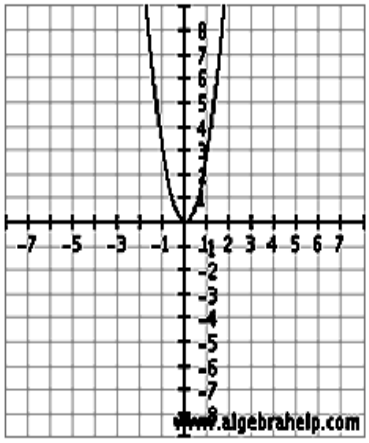
$y = -\frac{1}{3}x^2$



$y = 4x^2$

2. **State the vertex and the equation for each of the following graphs.**

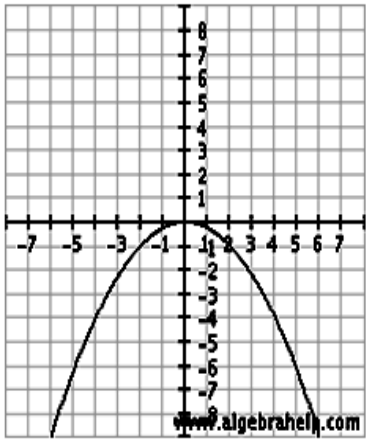
(a)



vertex = (0,0)

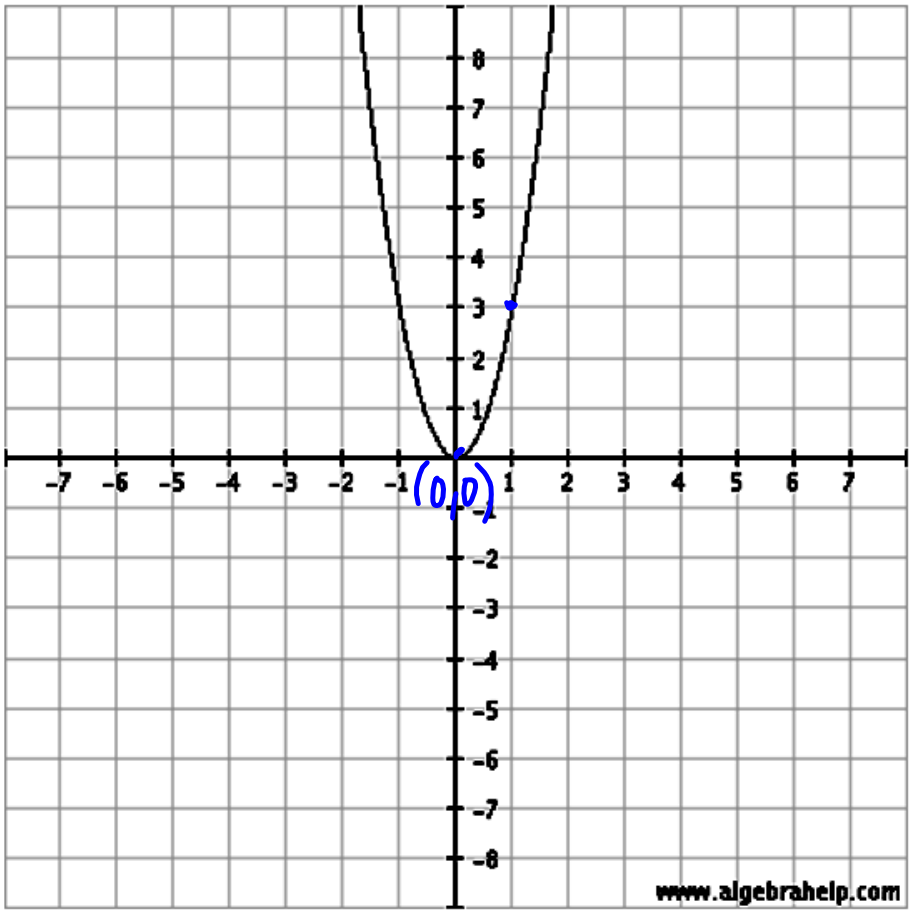
$y = 3x^2$

(b)



vertex = (0,0)

$y = -\frac{1}{4}x^2$

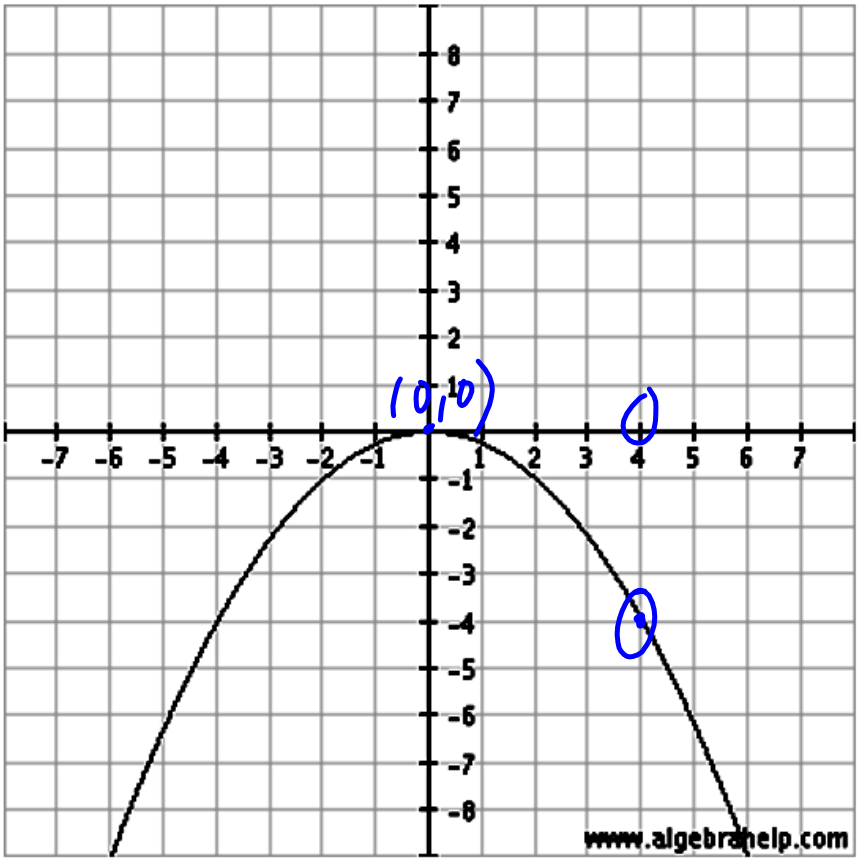


$y = 3x^2$

$x^2 = 1$

$3x^2 = ?$

x	x^2	
0	0	0
1	1	3
2	4	



$y = -\frac{1}{4}x^2$

x	x^2	
0	0	
1	1	
2	4	(1)
3	9	
4	16	(4)