Worksheet 6-7: Effects of Changing Variables on Compound Interest

PV

Investigation 1: Compare Different Interest Rates and Final Amount

Jake has saved $2500 and is considering three investment alternatives for 6 years.

Investment Plan A: at **10%**, compounded annually
Investment Plan B: at **8.5%**, compounded annually
Investment Plan C: at **7%**, compounded annually

$A = P(1+i)^n$

(a) Which investment plan gives the most return?
(b) Which investment plan gives the least return?

Plan A:

$A = 2500(1+0.1)^6$

$= 4428.90$

Most

Plan B:

$A = 2500(1+0.085)^6$

$= 4078.67$

Plan C:

$A = 2500(1+0.07)^6$

$= 3751.83$

Least

FV

Investigation 2: Compare Different Interest Rates and Present Value

Sandy needs $5000 in 4 years to pay college. She has three GIC options:

GIC Option A: at **9%**, compounded quarterly
GIC Option B: at **6%**, compounded quarterly
GIC Option C: at **3%**, compounded quarterly

$P = A(1+i)^{-n}$

(a) Which option requires the biggest principal invested?
(b) Which option requires the smallest principal invested?

GIC A

$P = 5000\left(1+\frac{0.09}{4}\right)^{-16}$

$= 3502.33$

smallest

GIC B

$P = 5000\left(1+\frac{0.06}{4}\right)^{-16}$

$= 3940.16$

GIC C

$P = 5000\left(1+\frac{0.03}{4}\right)^{-16}$

$= 4436.59$

biggest

Conclusion:
Interest rate affects the final amount and the present value:
The greater the interest rate, the greater the final amount.
The greater the interest rate, the smaller the present value, or principal required.

Investigation 3: Compare Different Term Lengths and Final Amount
Vince has saved $2000 and is considering three term deposit alternatives.

Term Deposit A: at 5%, compounded semi-annually for **1 years** N = 2
Term Deposit B: at 5%, compounded semi-annually for **3 years**
Term Deposit C: at 5%, compounded semi-annually for **5 years**

(a) Which term deposit alternative earns the most interest?
(b) Which term deposit alternative earns the least interest?

(A) $A = 2000\left(1+\frac{0.05}{2}\right)^{2}$ $= 2101.25$ Least

(B) $A = 2000\left(1+\frac{0.05}{2}\right)^{6}$ $= 2319.39$

(C) $A = 2000\left(1+\frac{0.05}{2}\right)^{10}$ $= 2560.17$ Most

Investigation 4: Compare Different Term Lengths and Present Value
Pam needs $50 000 to buy an apartment later and is considering three investment options.

(a) Which of the following investment plans require the biggest principal invested?
(b) Which of the following investment plans require the smallest principal invested?

Investment Option A: at 6%, compounded monthly for **2 years**
Investment Option B: at 6%, compounded monthly for **4 years**
Investment Option C: at 6%, compounded monthly for **6 years**

(A) $P = 50000\left(1+\frac{0.06}{12}\right)^{-24}$ $= 44359.28$ biggest

(B) $P = 50000\left(1+\frac{0.06}{12}\right)^{-48}$ $= 39354.92$

(C) $P = 50000\left(1+\frac{0.06}{12}\right)^{-72}$ $= 34915.12$ Smallest

Conclusion:
Term length affects the final amount and the present value:
The longer the term length, the greater the final amount.
The longer the term length, the smaller the present value, or principal required.

Investigation 5: Compare Different Compounding Frequencies and Final Amount

Sharon has $3000, and she can choose among three 2-year GIC options.

GIC Option A: at 6%, compounded **semi-annually** $N = 2$
GIC Option B: at 6%, compounded **quarterly** $N = 4$
GIC Option C: at 6%, compounded **monthly** $N = 12$

(a) Which GIC option will give the most return?
(b) Which GIC option will give the least return?

(A) $A = 3000\left(1 + \frac{0.06}{2}\right)^{4}$ $= 3376.53$ least

(B) $A = 3000\left(1 + \frac{0.06}{4}\right)^{8}$ $= 3379.48$

(C) $A = 3000\left(1 + \frac{0.06}{12}\right)^{24}$ $= 3381.48$ most

Investigation 6: Compare Different Compounding Frequencies and Present Value

Khan wants to have $7000 in 4 years to buy a car. He is interested in three 4-year GICs.

GIC Option A: at 8%, compounded **annually**
GIC Option B: at 8%, compounded **semi-annually**
GIC Option C: at 8%, compounded **quarterly**

(a) Which option requires the biggest principal invested?
(b) Which option requires the smallest principal invested?

(A) $P = 7000(1 + 0.08)^{-4}$ $= 5145.21$ biggest

(B) $P = 7000\left(1 + \frac{0.08}{2}\right)^{-8}$ $= 5114.83$

(C) $P = 7000\left(1 + \frac{0.08}{4}\right)^{-16}$ $= 5099.12$ smallest

Conclusion:
Compounding frequency affects the final amount and the present value:
The more frequent the compounding, the greater the final amount.
The more frequent the compounding, the smaller the present value, or principal required.

1. Kabir plans to purchase a new car in three years and hopes to have $18 000 at that time.

   (a) Determine the principal that Kabir needs to invest today to have $18 000 after three years:

   Option 1: at 4.5% per year, compounded monthly

   Option 2: at 4.5% per year, compounded semi-annually

   (b) Which principal is less? Why?

2. Barb plans to invest $10 000 in a term deposit for two years. She has three choices.

   A: 6.8% per year, simple interest

   B: 6.2% per year, compounded semi-annually

   C: 6.0 % per year, compounded quarterly

   Which plan should she choose? Why?

3. Suppose you recently inherited $8000. You plan to use half the money now and invest the other half for at least three years. You have narrowed the investment down to three choices:

A: 3.25% per year, simple interest, cashable any time

B: 3.00% per year, compounded monthly, cashable after two years

C: 3.50 % per year, compounded semi-annually, cashable after four years

(a) Which plan earns the most interest after four years? Does that mean it is the best option for you? Justify your response.

(b) You have decided you will use the investment to make a down payment on a car in two and a half years. Which plan should you choose? Why?