Worksheet 6-6: Exponential Growth and Decay Modelling

Doubling: Doubling time is the time it takes for a population to double in size.

y-intercept

The relation for doubling is $P = P_0(2)^{\frac{t}{d}}$, where P represents the population,

P_0 represents the initial population, $= a$

t represents time

d represents the doubling time, and

the base "2" indicates doubling

1. A bacteria culture began with 7500 bacteria. Its growth can be modelled using the formula $N = 7500(2)^{\frac{t}{36}}$, where N is the number of bacteria after t hours.

(a) What is the doubling time?

36 hours

(b) How many bacteria are present after 36 hours?

$$N = 7500(2)^{\frac{36}{36}}$$
$$= 7500(2)^1 = 15000$$

15000 bacteria are present after 36 hours.

(c) How many bacteria are present after 72 hours? How does this relate to the doubling time?

$$N = 7500(2)^{\frac{72}{36}}$$
$$= 7500(2)^2$$
$$= 7500(4)$$
$$= 30000$$

30000 bacteria are present after 72 hours.

72 hours = 2 × 36 hours

It gets doubled twice

$7500 \times 2 \times 2 = 30000$

Half-Life: Half-life is the time it takes for a quantity to decay to half its original amount.

The relation for doubling is $M = M_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where M represents the final quantity,

Base → remaining amount $\left(\frac{1}{3}\right)$

M_0 represents the initial quantity,
t represents time
h represents the half-life, and
$\frac{2}{3}$ takenaway **the base "$\frac{1}{2}$" indicates half-life**

2. All living organisms contain a known concentration of 1 part per trillion parts of carbon-14. Carbon-14 is a radioactive element. It is used to date ancient artefacts because it has a half-life of about 5730 years after the organism dies. The formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$ is used to calculate the concentration, C, in parts per trillion, remaining n years after death.

(a) What is the initial concentration of carbon-14 as given in the formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$?

1 part per trillion

(b) What would be the concentration of carbon-14 in a piece of cloth (made from plant fibres) after 5730 years?

0.5 or $\frac{1}{2}$ part per trillion

(c) What would be the concentration of carbon-14 in an animal bone after 50 000 years? Round your answer to five decimal places.

$C = \left(\frac{1}{2}\right)^{\left(\frac{50000}{5730}\right)}$ 0.002361626466 $\boxed{E-03}$ → 2 decimal

$= 0.00236$

0.5 $\boxed{x^y}$ $\boxed{(}$ 50000 ÷ 5730 $\boxed{)}$ $\boxed{=}$

Lose 10% of 1 $\boxed{}$

Base = ~~90%~~ 0.9

$y = 0.9^x$

Lose $\frac{1}{3}$

Base = $\frac{2}{3}$ → $y = \left(\frac{2}{3}\right)^x$

20% depreciation per year

Original = \$5000

$y = 5000(1 - 0.2)^x$

$y = 5000(0.8)^x$

$1 + 0.13$

10% increase $1 + 0.1$

$y = 5000(1.1)^x$

3. *E. coli* is a very harmful type of bacteria that can be found in meat that is improperly stored or handled. The relation $N = N_0(2)^{\frac{t}{20}}$ estimates the number of *E. coli*, N, of an initial sample of N_0 bacteria after t minutes, at 37°C (body temperature), under optimal conditions.

(a) What is the doubling time of *E. coli*?

20 minutes

(b) If a sample of *E. coli* contains 5000 bacteria, how many will there be after 1 hour?

$N = 5000(2)^{\frac{60}{20}}$

$= 40000$

(c) If a sample of *E. coli* contains 1000 bacteria, how many will there be after 1 day?

1 day $= 1 \times 24 \times 60 = 1440$

$N = 1000(2)^{\frac{1440}{20}}$

$= 4.72 \times 10^{24}$

$4.722366483 \quad \boxed{24}$

10^{24}

There will be 4.72×10^{24} bacteria.

4. The deer population of a national park was 250 deer $\boxed{\text{12 years ago.}}$ Today, there are 500 deer. Assuming the deer population has experienced exponential growth, write a relation representing the size of the deer population in the park. Use your relation to project the deer population in 25 years.

$$P = P_0(2)^{\frac{t}{d}}$$

$d = 12$ years

$$P = 250(2)^{\frac{t}{12}}$$

$t = 12 + 25 = 37$

$$P = 250(2)^{\frac{37}{12}}$$

$$= 2119$$

There will be 2119 deer.

5. The relation $T = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$ can be used to determine the length of time, t, in hours, that milk of a certain fat content will remain fresh. T is the storage temperature, in degrees Celsius.

(a) What is the freshness half-life of milk?

10 hours

(b) How long will milk keep fresh at 22°C?

$$\frac{22}{190} = \frac{\cancel{190}\left(\frac{1}{2}\right)^{\frac{t}{10}}}{\cancel{190}}$$

$$\frac{22}{190} = \left(\frac{1}{2}\right)^{\frac{t}{10}}$$

$$0.116 = \left(\frac{1}{2}\right)^{\frac{t}{10}}$$

Trial + Error

$t=20$, $T=0.25$
$t=25$, $T=0.177$
$t=30$, $T=0.125$
$t=32$, $T=0.109$
$t=31$, $T=0.117$

It will take about 31 hours.

(c) How long will milk keep fresh at 4°C?

6. For his science project, Ranjit placed agar, a gel made from seaweed, in a petri dish and infected it with bacteria. The measurement of the growth ring was used to estimate the number of bacteria present. Ranjit measured the growth ring for two weeks. He constructed an exponential growth model using the function $N(t) = 5(1.2)^{\frac{t}{2}}$, where $N(t)$ represents the number of bacteria present, in thousands, and t represents the time, in days.

(a) What is the number of bacteria present at the start of the experiment?

5000 bacteria

(b) How many bacteria were present after one day?

$N(1) = 5(1.2)^{\frac{1}{2}}$

$= 5.477$

If round to 2 decimals:

$5.48 \times 1000 = 5480$

$\therefore$ There are 5477 bacteria.

(c) How many bacteria present after one week?

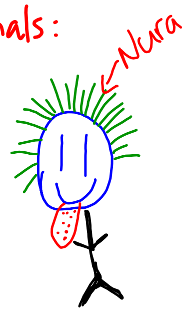
$N(7) = 5(1.2)^{\frac{7}{2}}$

$= 9.465$

If round to 2 decimals:

9.47×1000

$= 9470$ instead



$\therefore$ There are 9465 bacteria.

(d) How long does it take to double the number of bacteria present at the start of the experiment?

Double $= 5000 \times 2 = 10000$ $\therefore N(t) = 10$

$\frac{10}{5} = \frac{5(1.2)^{\frac{t}{2}}}{5}$

$2 = (1.2)^{\frac{t}{2}}$

Trial & Error

$t=5,\ N = 1.577$

$t=10,\ N = 2.488$

$t=8,\ N = 2.074 \rightarrow$ closest

$t=7,\ N = 1.893$

$\therefore$ It takes about 8 days.

(e) State the domain and range of the function for the growth of this bacterial culture.

Since it is an experiment for 2 weeks $\rightarrow$ 14 days, maximum for t is 14, maximum for N is N(14) which equals $N(14) = 5(1.2)^{\frac{14}{2}} = 17.92$ round up to 18

Domain $= \{t \in R,\ 0 \le t \le 14\}$

Range $= \{N(t) \in R,\ 5 \le N(t) \le 18\}$

7. Living organisms contain both carbon-12, which does not decay, and radioactive carbon-14. When the organism dies, the amount of carbon-14 decreases exponentially with a half-life of about 5730 years. One can measure the amount of carbon-14 left in an organism to estimate how long ago it died. The decay process can be modelled using the function $P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$, where P represents the percent of carbon-14 remaining and t represents the time, in years.

Start with 100%

(a) Bristlecone pine fossils found in the western United States show an age of about 6000 years using carbon dating. What percent of the original carbon-14 remains in these fossils? $P=?$

t

$$P = 100\left(\frac{1}{2}\right)^{\frac{6000}{5730}}$$
$$= 48.39$$

∴ 48.39% of original carbon-14 remains.

(b) Sediments deposited by glaciers during the last ice age covered and buried layers of peat. Carbon dating shows about 25% of the original carbon-14 remaining in the peat. Estimate the age of the glacier deposits to the nearest hundred years. $t=?$

P

$$\frac{25}{100} = \frac{100\left(\frac{1}{2}\right)^{\frac{t}{5730}}}{100}$$

$$0.25 = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

Trial + Error

Takes 5730 years $\rightarrow \frac{1}{2}$

×2 $\rightarrow \frac{1}{4} = 0.25$

Try $t = 5730 \times 2 = 11460$

$t = 11460$, $P = 0.25$ ✓

round to nearest hundreds

$\rightarrow 11500$

∴ It takes about 11500 years. round up NOT down!

(Sorry, answer key from textbook is wrong.)