Worksheet 6-5: Rates of Change/Growth

How do we identify linear, quadratic or exponential relations?

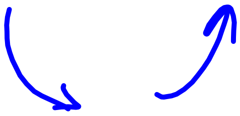
1. Classify by Equation:

y = 1 is still linear

- ☺ Linear: x is a first-degree variable (Exponent is 1).
- ☺ Quadratic: x is a second-degree variable (Exponent is 2).
- ☺ Exponential: x itself is the exponent.

2. Classify by Graph:

- ☺ Linear: the graph is a straight line.
- ☺ Quadratic: the graph is a parabola (U-shape).
- ☺ Exponential: the graph is an exponential curve (J-shaped).



3. Classify by Finite Differences:

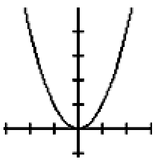
- ☺ Linear: first differences are constant.
- ☺ Quadratic: first differences increase by a constant value (adding). Second differences are constant.
- ☺ Exponential: first differences increase by a constant factor (multiplying), a common ratio.

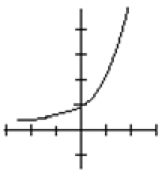
Check for Understanding:

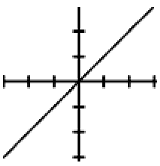
1. Without graphing, classify each of the following as linear, quadratic, or exponential growth.

(a) $3x - 4y = 12$	Linear	Quadratic	Exponential
(b) $y = 2x^2 + 3$	Linear	Quadratic	Exponential
(c) $y = \left(\frac{1}{5}\right)^x$	Linear	Quadratic	Exponential
(d) $y = 3(1.05)^x$	Linear	Quadratic	Exponential

2. Examine each graph and classify as linear, quadratic, or exponential growth.

(a)  Linear Quadratic Exponential

(b)  Linear Quadratic Exponential

(c)  Linear Quadratic Exponential

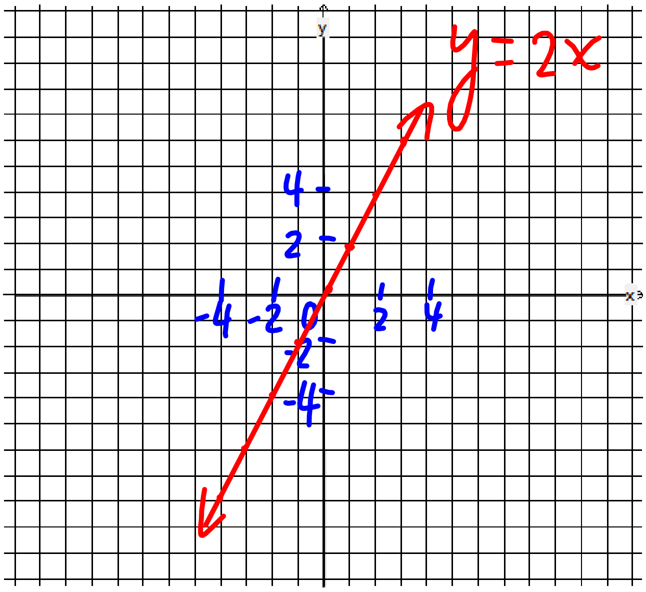
Investigation:

First Difference = New - Old

1. Graph $y = 2x$. Linear

x	$y = 2x$	First Difference
-3	$2(-3) = -6$	
-2	$2(-2) = -4$	$-4-(-6) = 2$
-1	$2(-1) = -2$	$-2-(-4) = 2$
0	$2(0) = 0$	$0-(-2) = 2$
1	$2(1) = 2$	$2-0 = 2$
2	$2(2) = 4$	$4-2 = 2$
3	$2(3) = 6$	$6-4 = 2$

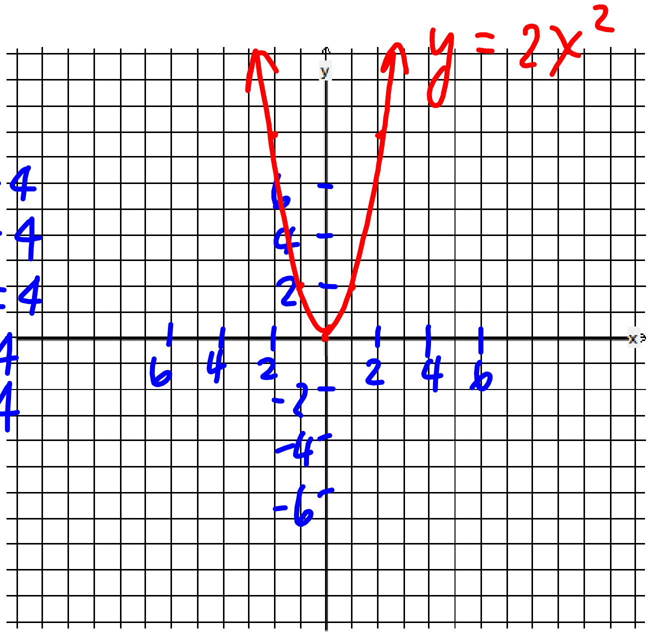
slope



2. Graph $y = 2x^2$.

x	$y = 2x^2$	First Difference	2nd Diff
-3	$2(-3)^2 = 18$		
-2	$2(-2)^2 = 8$	$8-18 = -10$	
-1	$2(-1)^2 = 2$	$2-8 = -6$	$-6-(-10) = 4$
0	$2(0)^2 = 0$	$0-2 = -2$	$-2-(-6) = 4$
1	$2(1)^2 = 2$	$2-0 = 2$	$2-(-2) = 4$
2	$2(2)^2 = 8$	$8-2 = 6$	$6-2 = 4$
3	$2(3)^2 = 18$	$18-8 = 10$	$10-6 = 4$

2nd differences are constant



3. Graph $y = 2^x$.

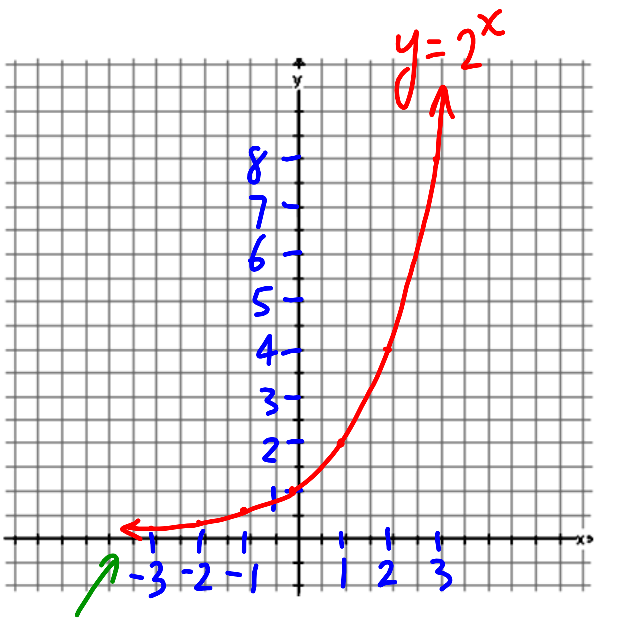
$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

x	$y = 2^x$	First Difference	Ratio
-3	$2^{-3} = \frac{1}{8}$		
-2	$2^{-2} = \frac{1}{4}$	$\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$	$\times 2$
-1	$2^{-1} = \frac{1}{2}$	$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$	$\times 2$
0	$2^{0} = 1$	$1 - \frac{1}{2} = \frac{1}{2}$	$\times 2$
1	$2^{1} = 2$	$2 - 1 = 1$	$\times 2$
2	$2^{2} = 4$	$4 - 2 = 2$	$\times 2$
3	$2^{3} = 8$	$8 - 4 = 4$	$\times 2$

Ratio $= \frac{\text{New}}{\text{Old}}$

$\frac{1}{4} \div \frac{1}{8}$

$= 1 \div 4 \div (1 \div 8)$

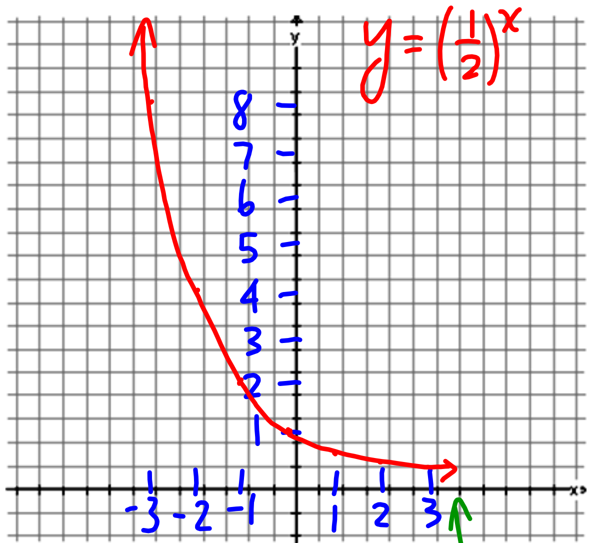


Approaching but never touching x-axis → horizontal asymptote is $\boxed{y=0}$

$\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = 2^3$

4. Graph $y = \left(\frac{1}{2}\right)^x$.

x	$y = \left(\frac{1}{2}\right)^x$	First Difference	Ratio
-3	$\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$		
-2	$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$	-4	$\times \frac{1}{2}$
-1	$\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$	-2	$\times \frac{1}{2}$
0	$\left(\frac{1}{2}\right)^{0} = 1$	-1	$\times \frac{1}{2}$
1	$\left(\frac{1}{2}\right)^{1} = \frac{1}{2}$	$-\frac{1}{2}$	$\times \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$	$-\frac{1}{4}$	$\times \frac{1}{2}$
3	$\left(\frac{1}{2}\right)^{3} = \frac{1}{8}$	$-\frac{1}{8}$	$\times \frac{1}{2}$



Reflection of $y = 2^x$ in the y-axis

approaching but never touching x-axis → Horizontal asymptote is $\boxed{y=0}$

5. **Conclusions:**

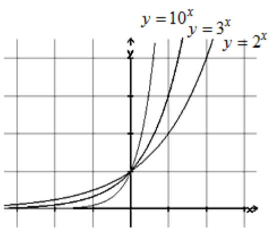
☺ $y = 2x$ is a(n) linear relation because x is a first-degree variable.
So, the graph is a(n) straight line and it represents linear growth.

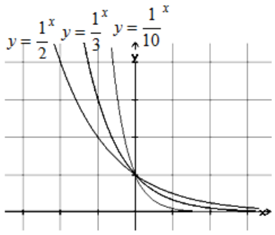
☺ $y = 2x^2$ is a(n) quadratic relation because x is a second-degree variable.
So, the graph is a(n) parabola and it represents quadratic growth.

☺ $y = 2^x$ is a(n) exponential relation because x is an exponent.
So, the graph is a(n) J-shaped curve that rises up to the right and it represents exponential growth.

☺ $y = \left(\frac{1}{2}\right)^x$ is a(n) exponential relation because x is an exponent.
So, the graph is a(n) J-shaped curve that falls down to the right and it represents exponential decay.

Note:
For exponential relation $y = b^x$, the curve gets closer to the y-axis as b increases when $b > 1$ or as b decreases (denominator of b increases) when $0 < b < 1$ (a fraction or a decimal less than 1).





Check for Understanding:
Create the next diagram following the given pattern, and determine what type of relation is represented.

Diagram 1 Diagram 2 Diagram 3 Diagram 4

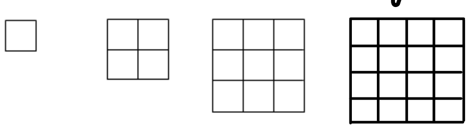


Diagram Number	Number of squares	1st Diff	2nd Diff
1	1		
2	4	3	
3	9	5	2
4	16	7	2

Constant

Since second differences are constant, the relation between diagram number and the number of squares is quadratic.

$$(0.0625)^{\frac{3}{4}}$$

$$= \left(\sqrt[4]{\frac{625}{10000}}\right)^3$$

$$= \left(\frac{5}{10}\right)^3$$

$$= \frac{125}{1000}$$

$$0.1 = \frac{1}{10}$$