Mini-Test 6-1 on WS 6-1, 6-2 and 6-3

Tuesday May 12

#9 #6 #1

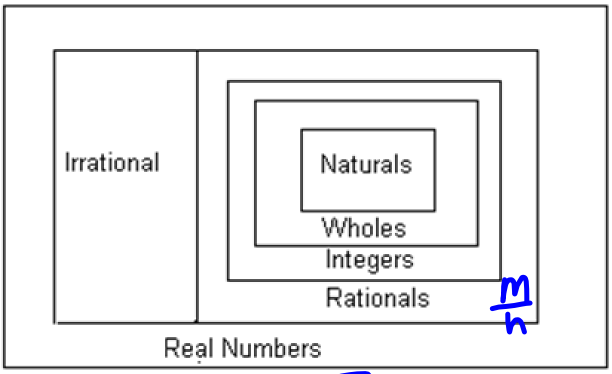
#7 #8

#3 #2

#4 #5

Worksheet 6-3: Rational Exponents

Real Number System:
Natural numbers
Whole numbers
Integers
Rational numbers



$\frac{1}{9}$

Rational Exponent Investigation:

$\sqrt[2]{\ }$ $\sqrt[x]{\ }$ $\sqrt[\square]{\ }$

$9 = 9^1$ $\frac{1}{2} \times 2 = 1$

(a) $9 = \left(9^{\frac{1}{2}}\right)^2$	**(b)** $25 = \left(25^{\frac{1}{2}}\right)^2$	**(c)** $8 = \left(8^{\frac{1}{3}}\right)^3$	**(d)** $16 = \left(16^{\frac{1}{4}}\right)^4$
but $9 = (3)^2$	but $25 = (5)^2$	but $8 = (2)^3$	but $16 = (2)^4$
so $\left(9^{\frac{1}{2}}\right)^2 = 3^2$	so $\left(25^{\frac{1}{2}}\right)^2 = 5^2$	so $\left(8^{\frac{1}{3}}\right)^3 = 2^3$	so $\left(16^{\frac{1}{4}}\right)^4 = 2^4$
and $9^{\frac{1}{2}} = 3$	and $25^{\frac{1}{2}} = 5$	and $8^{\frac{1}{3}} = 2$	and $16^{\frac{1}{4}} = 2$
so $9^{\frac{1}{2}} = \sqrt{9}$	so $25^{\frac{1}{2}} = \sqrt{25}$	so $8^{\frac{1}{3}} = \sqrt[3]{8}$	so $16^{\frac{1}{4}} = \sqrt[4]{16}$

e.g. $36^{\frac{1}{2}}$, $81^{\frac{1}{4}}$, $1000^{\frac{1}{3}}$, $128^{\frac{1}{7}}$ write in radical form then evaluate.

Power form: $x^{\frac{1}{n}}$ — exponent: $\frac{1}{n}$; base: x

Radical form: $\sqrt[n]{x}$ — index: n; radical sign: $\sqrt{\ }$; radicand: x

- The index tells us what kind of root it is.

A **square root** has an index of **2**.
A **cube root** has an index of **3**.
A **fourth root** has an index of **4**, and so on.

$a^{\frac{1}{n}} = \sqrt[n]{a}$, where n is a natural number. $a^m \times a^{\frac{1}{n}}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$, where m, n are natural numbers.

If n is even, then $a \geq 0$. i.e. a cannot be negative.
If n is odd, then a can be any real number.

To evaluate $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$, take the n^{th} root of a or find the value of a real number which is to be multiplied by itself n times to equal a.

To evaluate $a^{\frac{m}{n}}$, either take the n^{th} root of a, then raise the result to the m^{th} power, or raise a to the m^{th} power, then take the n^{th} root.

Practice: $\sqrt[3]{-8} = -2$ $\quad \sqrt[2]{4} = 2$

1. Evaluate, where possible. Verify your answers with a calculator.

(a) $100^{\frac{1}{2}}$ $= (10^2)^{\frac{1}{2}} = 10$

(b) $1000^{\frac{1}{3}}$ $= (10^3)^{\frac{1}{3}} = 10$

(c) $32^{\frac{1}{5}}$ $= (2^5)^{\frac{1}{5}}$

(d) $\sqrt[4]{0.0001}$ $= (0.1^4)^{\frac{1}{4}} = 0.1$

(e) $(-100)^{\frac{1}{2}}$ $= \sqrt[2]{-100}$ Doesn't exist

(f) $\sqrt[3]{-1000}$ $= ((-10)^3)^{\frac{1}{3}} = -10$

(g) $(-32)^{\frac{1}{5}}$ $= \sqrt[5]{-32} = ((-2)^5)^{\frac{1}{5}} = -2$

(h) $(-0.0001)^{\frac{1}{4}}$ $= \sqrt[4]{-0.0001}$ Doesn't exist

$9 = 3\times3$ or -3×-3

-9

$\sqrt[3]{8} \Rightarrow \boxed{x^3 = 8}$

2. Evaluate each of the following. Check your answers with a calculator. $\boxed{2/3}$

(a) $16^{\frac{3}{4}}$ $\quad 16^{\boxed{3/4}}$ $= (16^{\frac{1}{4}})^3 = 2^3 = 8$

(b) $36^{\frac{3}{2}}$ $\quad 36^{\boxed{3/2}}$ $= (36^{\frac{1}{2}})^3 = 6^3 = 216$

(c) $(-64)^{\frac{2}{3}}$ $\quad (-64)$ $= (-64^{\frac{1}{3}})^2 = (-4)^2 = 16$

(d) $0.008^{\frac{4}{3}}$ $= \left(\frac{8}{1000}\right)^{\frac{4}{3}} = \left(\left(\frac{8}{1000}\right)^{\frac{1}{3}}\right)^4 = \left(\frac{2}{10}\right)^4 = \frac{16}{10000}$

(e) $16^{-\frac{1}{2}}$

(f) $\left(\frac{1}{25}\right)^{-\frac{1}{2}}$

(g) $\left(\frac{16}{25}\right)^{-\frac{1}{2}}$ $= \left(\frac{25}{16}\right)^{\frac{1}{2}} = \frac{25^{\frac{1}{2}}}{16^{\frac{1}{2}}} = \frac{5}{4}$

(h) $\left(\frac{64}{27}\right)^{-\frac{2}{3}}$ $= \left(\frac{27}{64}\right)^{\frac{2}{3}} = \frac{27^{\frac{2}{3}}}{64^{\frac{2}{3}}} = \frac{(\sqrt[3]{27})^2}{(\sqrt[3]{64})^2} = \frac{3^2}{4^2} = \frac{9}{16}$

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt[2]{9}} = \frac{1}{3}$$