Worksheet 2-2: Theoretical Probability

Theoretical probability is another measure of the likelihood of an event. It is the ratio of the number of successful outcomes and the total number of possible outcomes.

Theoretical Probability of An Event
$P(\text{Event A}) = \frac{\text{Number of Successful Outcomes}}{\text{Total Number of Possible Outcomes}}$

To calculate the theoretical probability, all outcomes must be equally likely.

"Equally likely" means the same chance of occurring because the conditions are fair.

For example, in the toss of a fair coin, the chances of getting heads or tails are equally likely.

Experiment	Event	Total Number of Possible Outcomes	Number of Successful Outcomes	Theoretical Probability
Toss a coin	Turning up heads	2 H T	1	$\frac{1}{2}$
Roll one die	Turning up 4 even #s	6	1 3	$\frac{1}{6}$ $\frac{1}{2}$
Draw a playing card	Drawing an ace A	52	4	$\frac{1}{13}$ $\frac{4}{52} = \frac{1}{13}$

1. A standard deck of playing cards has 52 cards, 13 of each suit. If one card is drawn from the deck, find the probability of each event.

(a) a heart

$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$

(b) a heart, a club or a jack

13 13 2 – spade + diamond

$P(\text{a heart, club, or jack}) = \frac{28 \div 4}{52 \div 4} = \frac{7}{13}$

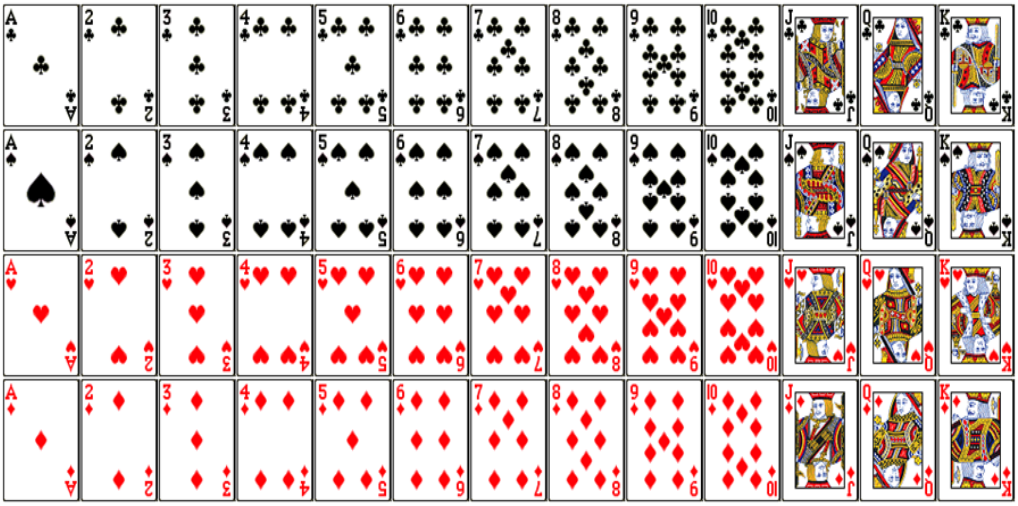
(b) a black diamond

$P(\text{black diamond}) = \frac{0}{52} = 0$

(d) a heart, a club, a spade, or a diamond

13 13 13 13

$P(\text{a heart, club, spade or diamond}) = \frac{52}{52} = 1$



2. What is the probability of rolling doubles with a pair of dice?

1st die (6) 2nd die (6)

Total possible outcomes = 6×6 ← multiply

$= 36$

Doubles:

(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

$P(\text{doubles}) = \frac{6}{36}$

$= \frac{1}{6}$

3. Suppose you roll two six-sided dice. Find the theoretical probability of rolling each sum. Express each answer as a fraction in lowest terms.

(a) 2

$P(\text{sum}=2) = \frac{1}{36}$

(b) 11

(6,5) (5,6)

$P(\text{sum}=11) = \frac{2}{36} = \frac{1}{18}$

(c) 7 (6,1)(1,6)(3,4)(4,3)(5,2)(2,5)

$P(\text{sum}=7) = \frac{6}{36} = \frac{1}{6}$

(d) not 7

$P(\text{sum}\neq 7) = \frac{36-6}{36} = \frac{30}{36} = \frac{5}{6}$

$1 - \frac{1}{6} = \frac{5}{6}$

(e) a sum greater than 5

$P(\text{sum} > 5) \rightarrow \text{sum} = 6, 7, 8, 9, 10, 11, 12$

$\rightarrow \text{sum} \leq 5 \rightarrow 2, 3, 4, 5$

$P(\text{sum} \leq 5) + P(\text{sum} > 5) = 1$

(1,1)
(2,1)(1,2)
(2,2)(1,3)
(3,1)
(2,3)(3,2)
(1,4)(4,1)

$P(\text{sum} > 5) = 1 - P(\text{sum} \leq 5)$

$= 1 - \frac{10}{36}$

$= 1 - \frac{5}{18}$

$= \frac{13}{18}$

4. During a game of musical chairs, 10 people walk around eight chairs waiting for the music to stop. Find the probability of a person not getting a chair.

$P(\text{no chair}) = \frac{2}{10}$

$= \frac{1}{5}$

∴ The probability of a person not getting a chair is $\frac{1}{5}$.

5. Suppose you roll two six-sided dice.

(a) Explain why the probability of rolling a sum of 14 is 0.

Since the largest on a six-sided die is 6, the largest sum of 2 dice is 12.

It is impossible to roll a sum of 14, so the probability of rolling a sum of 14 with 2 six-sided dice is 0.

(b) Explain why the probability of rolling a sum from 2 to 12 is 1.

Any sum from 2 to 12 can be rolled by 2 six-sided dice. In this case, the number of successful outcomes is 36 which is the same as total possible outcomes.

$\frac{36}{36}$ is 1, so the probability of rolling a sum from 2 to 12 is 1.

6. A card is randomly selected from a standard deck of cards. Write the theoretical probability of each event as a fraction in lowest terms.

(a) a red king

$P(\text{red king}) = \frac{2}{52} = \frac{1}{26}$

1 king of heart and 1 king of diamond } 2

(b) a face card

$P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$

4 king
4 Queen
4 Jack
} 12

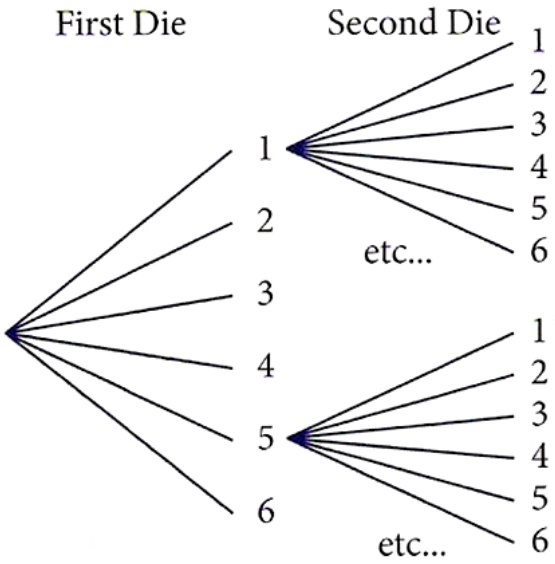
Get Used to the Language or Special Terms for Probability

① A red king card
red & king
AND 2

② Red or King card
Red or King
26 + 4 − 2
Double counting
26 + 2

③ Red or Black
26 + 26

Ways to show sum of two dice:



Ways to show sum of two dice:

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Ways to show sum of two dice: