Worksheet 1-5: Quadratic Functions $y = ax^2 + bx + c$ where $a \neq 0$

Quadratic Functions:

A quadratic function is a 2nd degree function that involves not only an x-term and a constant term like a linear function $y = mx + b$, but it also has an x^2 term.

Note: **The x^2 term tells that it is a quadratic function only when its exponent is the highest.

> **A quadratic function is a function that can be described by an equation of the form $y = ax^2 + bx + c$ where $a \neq 0$.**

1. For the following quadratic equations in the form $y = ax^2 + bx + c$, state the value of a, b, and c.

(a) $y = 3x^2 + 2x - 1$ (b) $y = -4x^2 - 5x + 9$ (c) $y = 2x^2$

$a=3, b=2, c=-1$ $a=-4, b=-5, c=9$ $a=2, b=0, c=0$

2. Which of the following is a quadratic function?

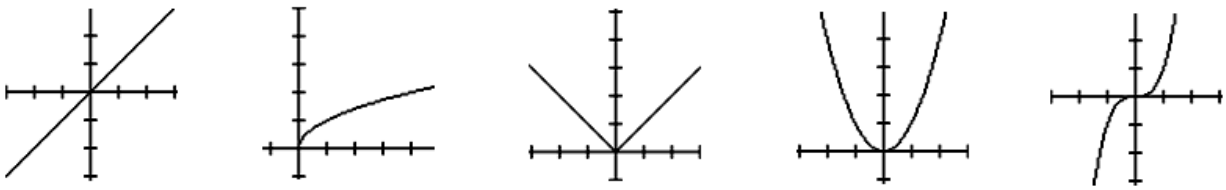
$y = x^2$ $y = 3x$ $y = 2^x$ $y = -x^2 + 7x - 1$

$y = 2x + 4$ $y = x^3 - x^2$ $y = 9x^2 + 3x - 1$ $y = x + x^2$

Parabola: The Graph of a Quadratic Function

The graph of a quadratic function is a **U-shaped** curve called a parabola.
The base equation $y = x^2$ gives the basic parabola.

3. Which of the following is a parabola?



4. How do you know that a parabola is a function?

It passes the vertical line test.

Second Differences of a Quadratic Function are Constant

For linear functions, first differences are constant (first degree equations).
For quadratic functions, second differences are constant (second degree equations).

5. Which of the following table of values represent a quadratic function?

(a)

x	y	1st Diff.
0	3	
1	6	+3
2	9	+3
3	12	+3

+1, +1, +1 → 3

This is linear

(b)

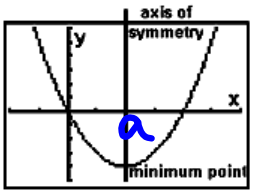
x	y	1st Diff	2nd Diff
0	1		
1	6	5	
2	9	3	-2
3	10	1	-2

+1, +1, +1

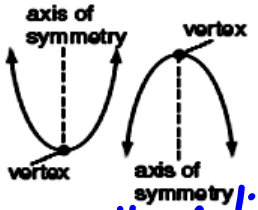
This is quadratic.

Properties of a Quadratic Function:

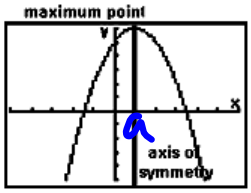
- **Vertex:** A quadratic function has a maximum or a minimum value at its vertex (turning point).
  When the curve opens upward, the vertex gives the minimum value.
  When the curve opens downward, the vertex gives the maximum value.



x = a



vertical line



x = a

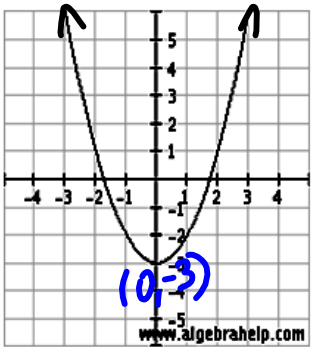
- **Axis of Symmetry:** A quadratic function is symmetrical about the vertical line that passes through the vertex. This line is called the axis of symmetry.

6. For the following parabolas,

(a) State the ordered pair of the vertex.
(b) Does the curve open upward or downward?
(c) State the maximum or minimum value.
(d) State the equation of the line of symmetry: **x = x-coordinate of the vertex.**

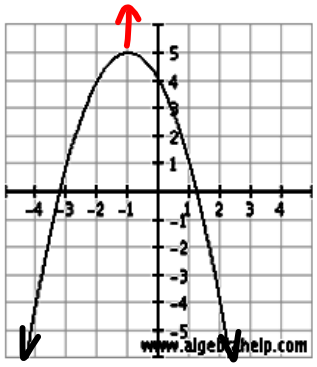
(h, k)
x = h

(i)



(a) vertex = (0, -3)
(b) upward
(c) minimum value = -3
D = {x ∈ R}
R = {y ∈ R | y ≥ -3}
(d) x = 0

(ii)

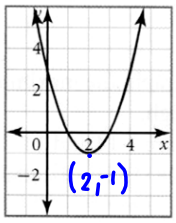


(a) vertex = (-1, 5)
(b) downward
(c) maximum value = 5
D = {x ∈ R}
R = {y ∈ R | y ≤ 5}
(d) x = -1

For each of the given graphs of quadratic functions:

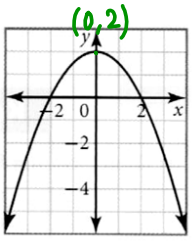
(a) State the coordinates of the vertex.
(b) Does the parabola open upward or downward?
(c) State the maximum or minimum y-value.
(d) State the equation for the axis of symmetry.
(e) State the x-intercepts if they exist. What is the y-coordinate of each x-intercept?
(f) State the y-intercept if it exists. What is the x-coordinate of the y-intercept?
(g) State the domain and range.

7.



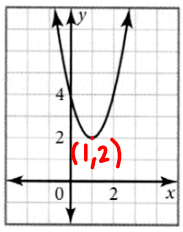
(a) Vertex = (2,-1)
(b) opens upward
(c) minimum value = -1
(d) $x = 2$
(e) x-intercepts are 1 and 3 whose y-coordinates are 0.
(f) y-intercept is 3 whose x-coordinate is 0.
(g) Domain = $\{x \in R\}$
Range = $\{y \in R \mid y \geq -1\}$

8.



(a) vertex = (0,2)
(b) opens downard
(c) maximum value = 2
(d) $x = 0$
(e) x-intercepts are -2 and 2 whose y-coordinates are 0.
(f) y-intercept is 2 whose x-coordinate is 0.
(g) Domain = $\{x \in R\}$
Range = $\{y \in R \mid y \leq 2\}$

9.

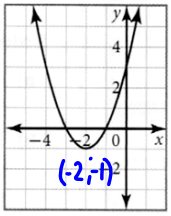


(a) vertex = (1,2)
(b) opens upward
(c) minimum value = 2
(d) $x = 1$
(e) There is no x-intercept
(x-intercept = none NOT 0)
(f) y-intercept is 4 whose x-coordinate is 0.
(g) Domain = $\{x \in R\}$
Range = $\{y \in R \mid y \geq 2\}$

For each of the given graphs of quadratic functions:

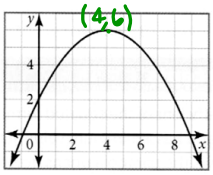
(a) State the coordinates of the vertex.
(b) Does the parabola open upward or downward?
(c) State the maximum or minimum y-value.
(d) State the equation for the axis of symmetry.
(e) State the x-intercepts if they exist. What is the y-coordinate of each x-intercept?
(f) State the y-intercept if it exists. What is the x-coordinate of the y-intercept?
(g) State the domain and range.

10.



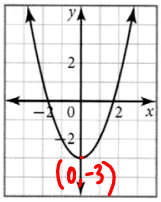
(a) vertex = $(-2,-1)$
(b) opens upward
(c) minimum value = -1
(d) $x = -2$
(e) x-intercepts are -1 and -3 whose y-coordinates are 0.
(f) y-intercept is 3 whose x-coordinate is 0.
(g) Domain = $\{x \in R\}$
Range = $\{y \in R \mid y \geq -1\}$

11.



(a) vertex = $(4,6)$
(b) opens downward
(c) maximum value = 6
(d) $x = 4$
(e) x-intercepts are -1 and 9 whose y-coordinates are 0.
(f) y-intercept is 2 whose x-coordinate is 0.
(g) Domain = $\{x \in R\}$
Range = $\{y \in R \mid y \leq 6\}$

12.



(a) vertex = $(0,-3)$
(b) opens upward
(c) minimum value = -3
(d) $x = 0$
(e) x-intercepts are -1.75 and 1.75 whose y-coordinates are 0.
(f) y-intercept is -3 whose x-coordinate is 0.
(g) Domain = $\{x \in R\}$
Range = $\{y \in R \mid y \geq -3\}$