

Practice Test 5: Exponential Relations

K: _____	C: _____	A: _____	T: _____
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PART A: Multiple Choice Questions

Instructions: Circle the English letter of the best answer. Circle one and **ONLY** one answer.

Knowledge/Thinking:

1. Which of these relations is exponential?

- (a) $y = 0.5x$ (b) $y = 3x^2$ (c) $y = 2(x)^3$ (d) $y = 5x^{\frac{1}{2}}$ (e) $y = 3^x$

2. 2^{-4} is:

- (a) -8 (b) -2^4 (c) $\frac{1}{16}$ (d) -16 (e) $-\frac{1}{8}$

3. -3.5^0 is:

- (a) -3.5 (b) 3.5 (c) 1 (d) -1 (e) 0

4. Evaluate $2.58^{-1.4}$ to 2 decimal places, the answer is:

- (a) 0.26 (b) 0.27 (c) -0.27 (d) 3.61 (e) -3.61

5. $100(2.5)^{\frac{10}{5}}$ is:

- (a) 62500 (b) 625 (c) 190735 (d) 1907 (e) 500

6. A certain type of bacteria doubles every 8 hours. A culture begins with 30 000 bacteria. Which of the following formula best models the growth of the bacteria in the culture?

- (a) $P = 30000(2)^x$ (b) $P = 30000(2)^{8x}$ (c) $P = 30000(8)^x$ (d) $P = 30000(2)^{\frac{8}{x}}$ (e) $P = 30000(2)^{\frac{x}{8}}$

7. Carbon-14 is a radioactive element, and its decay can be modelled by the formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$.

What is the half-life of Carbon-14?

- (a) 5730 (b) 0.5 (c) 14 (d) 2865 (e) unknown

8. Which of these relations is exponential?

(a)

x	y
1	2
2	4
3	6
4	8

(b)

x	y
2	4
4	16
6	36
8	64

(c)

x	y
0	32
1	8
2	2
3	1

(d)

x	y
0	-8
1	16
2	-32
3	64

Part B: Full Solution Questions

Instructions: Show all steps for full mark.

Provide answer statements in complete English sentences where applicable.

Knowledge:

1. Write each as a single power with positive exponents, then evaluate as integers or fractions.

(a) $\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{-4}$ [K: 3]

(b) $((-3)^2)^{-1}$ [K: 3]

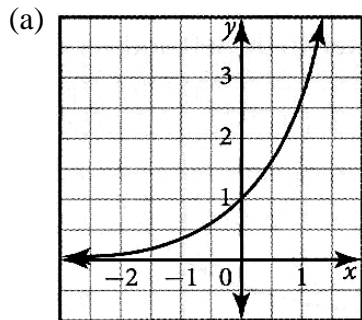
(c) $\frac{10^{20} \times 10^{-23}}{10^{-3}}$ [K: 3]

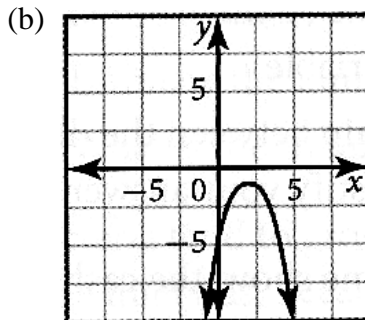
(d) $\left(\frac{2}{3}\right)^{-2}$ [K: 2]

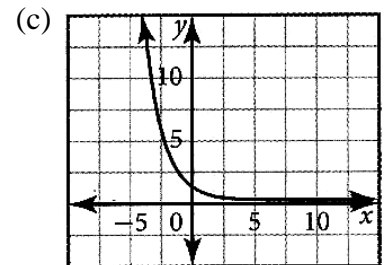
(e) $\frac{4^3 \times 4 \times 4^5}{(4^2)^3}$ [K: 4]

(f) $(-5)^{-4} \times (-5)^5 \div (-5)^{-1}$ [K: 3]

2. Classify each as exponential growth, exponential decay, quadratic growth or linear growth. [K: 3]







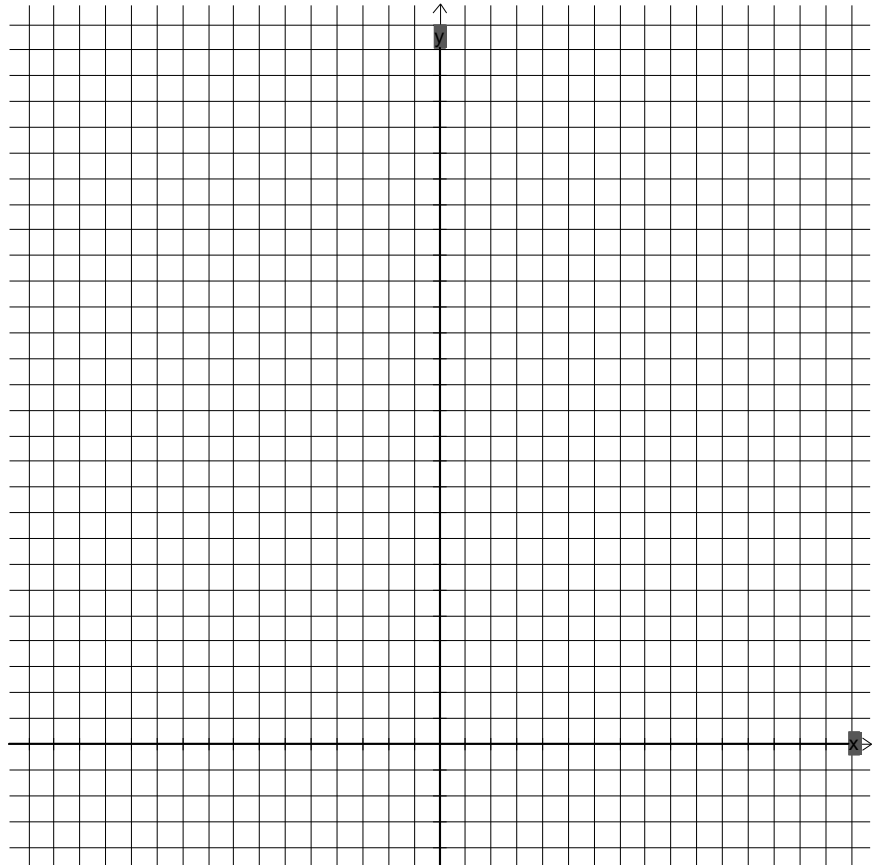
Communication:

3. For $y = \left(\frac{1}{2}\right)^x$ and $y = 2\left(\frac{1}{2}\right)^x$,

(a) graph both relation on the same axes. [C: 5]

x	$y = \left(\frac{1}{2}\right)^x$
-3	
-2	
-1	
0	
1	
2	
3	

x	$y = 2\left(\frac{1}{2}\right)^x$
-3	
-2	
-1	
0	
1	
2	
3	



(b) describe the differences and similarities of the two relations. [C: 4]

4. Which model (linear, quadratic, or exponential) would best describe each situation? Why? [C: 6]

(a) an airplane slowing down by half of its speed for every minute that elapses

(b) the height of a bottle falling down from the top of a building

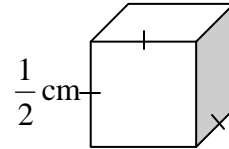
(c) the deer population in a national park doubling every year

(d) a marathon runner speeding up by 2 km/h each second

5. Johnny was asked to solve a problem involving an exponential relation. He was given the equation of an exponential relation: $y = 224(1.075)^x$, and ask to find the value of y when $x = 10$. Describe a situation that can be modelled by this equation and the answer to be determined. [C: 4]

Application:

6. The side length of a cube is $\frac{1}{2}$ cm. Write the volume of the cube as a single power, and then evaluate. [A: 2]



7. A town's racoon population is growing exponentially. The expected population can be estimated using the relation $P = 1250(1.013)^n$, where P is the population and n is the number of years.

(a) What is the current racoon population? [A: 1]

(b) What is the growth factor for the relation? [A: 1]

(c) What is the yearly growth rate of the racoon population? Write as a percent. [A: 1]

(d) What is the expected population in 5 years? [A: 2]

(e) How long does it take the racoon population to be doubled? [A: 3]

8. The remaining mass of a drug in a person's bloodstream is modelled by $M = 500\left(\frac{1}{2}\right)^{\frac{t}{2}}$, where M is the remaining mass in milligrams, and t is the time, in hours, that the drug is in the bloodstream.

(a) What is the half-life of the drug? [A: 1]

(b) What was the dosage of the drug? [A: 1]

(c) What will be the concentration of the drug in the bloodstream after 2 hours? [A: 2]

(d) What will be the concentration of the drug in the bloodstream after 6 hours? [A: 2]

(e) How long does it take for the concentration of the drug in the bloodstream to reduce to 25%? [A: 2]

Thinking:

9. The deer population of a national park was 200 deer 10 years ago. Today, there are 400 deer. Assuming the deer population has experienced exponential growth, write a relation representing the size of the deer population in the park. Use your relation to project the deer population in 25 years. [T: 4]

10. From 1994 to 2004, average personal incomes grew in Canada according to the relation $I = I_0(1.041)^n$, where I is the resulting income, I_0 is the initial income, and n is the number of years of growth.

(a) If a person’s income was \$34 000 in 1994, what would it be in 2004? [T: 2]

(b) If a person’s income was \$50 000 in 1996, what would it be in 2003? [T: 3]

(c) What was the average yearly rate of growth from 1994 to 2004? Write as a percent. [T: 1]

11. The amplitude of a pendulum over a 40-s period is shown in the table. Write an exponential relation to model the situation. [T: 4]

Time (s)	0	10	20	30	40
Amplitude (cm)	80.0	40.0	20.0	10.0	5.0

Answers:**Part A:**

1. e; 2. c; 3. d; 4. b; 5. b; 6. e; 7. a; 8. d.

Part B:

1. (a) 2^2 , 4, (b) $\frac{1}{(-3)^2}$, $\frac{1}{9}$, (c) 10^0 , 1, (d) $\left(\frac{3}{2}\right)^2$, $\frac{9}{4}$, (e) 4^3 , 64, (f) $(-5)^2$, 25;
2. (a) exponential growth, (b) quadratic growth, (c) exponential decay;
3. (b) Both relations have the same shape. Both decrease more rapidly and then more slowly to almost horizontal but never touch the x -axis as x increases. The two relations have different y -intercepts (1 for $y = \left(\frac{1}{2}\right)^x$ and 2 for $y = 2\left(\frac{1}{2}\right)^x$);
4. (a) exponential, because speed decreases by the same rate over equal time periods,
(b) quadratic, because gravity involves quadratics,
(c) exponential, because population increases by the same rate over equal time periods,
(d) linear, because distance increases by the same amount over equal time periods;
5. Answer may vary but must have 224 as the initial amount, a growth factor of 1.075 or a growth rate of 7.5%, and 10 as the number of changes. For example, there are currently 224 swans in the Toronto Zoo. If the swan population grows at a rate of 7.5% per year, how many swans will there be after 10 years?
6. $\left(\frac{1}{2}\right)^3$, $\frac{1}{8}$ cm³;
7. (a) 1250, (b) 1.013, (c) 1.3%, (d) 1333, (e) 54 years;
8. (a) 2 hours, (b) 500 mg, (c) 250 mg, (d) 62.5 mg, (e) 4 hours;
9. (a) $P = 200(2)^{\frac{x}{10}}$, (b) 2263;
10. (a) \$50 814, (b) \$66 241, (c) 4.1%;
11. $y = 80\left(\frac{1}{2}\right)^{\frac{x}{10}}$.