

Worksheet 5-8: Doubling and Half-Life

Doubling: Doubling time is the time it takes for a population to double in size.

The relation for doubling is $P = P_0(2)^{\frac{t}{d}}$, where P represents the population,

P_0 represents the initial population,

t represents time

d represents the doubling time, and

the base “2” indicates doubling

1. A bacteria culture began with 7500 bacteria. Its growth can be modelled using the formula

$N = 7500(2)^{\frac{t}{36}}$, where N is the number of bacteria after t hours.

(a) What is the doubling time?

(b) How many bacteria are present after 36 hours?

(c) How many bacteria are present after 72 hours? How does this relate to the doubling time?

Half-Life: Half-life is the time it takes for a quantity to decay to half its original amount.

The relation for doubling is $M = M_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$, where M represents the final quantity,

M_0 represents the initial quantity,

t represents time

h represents the half-life, and

the base “ $\frac{1}{2}$ ” indicates half-life

2. All living organisms contain a known concentration of 1 part per trillion parts of carbon-14. Carbon-14 is a radioactive element. It is used to date ancient artefacts because it has a half-life of about 5730 years after the organism dies. The formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$ is used to calculate the concentration, C , in parts per trillion, remaining n years after death.

(a) What is the initial concentration of carbon-14 as given in the formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$?

(b) What would be the concentration of carbon-14 in a piece of cloth (made from plant fibres) after 5730 years?

(c) What would be the concentration of carbon-14 in an animal bone after 50 000 years? Round your answer to five decimal places.

3. *E. coli* is a very harmful type of bacteria that can be found in meat that is improperly stored or handled. The relation $N = N_0(2)^{\frac{t}{20}}$ estimates the number of *E. coli*, N , of an initial sample of N_0 bacteria after t minutes, at 37°C (body temperature), under optimal conditions.

(a) What is the doubling time of *E. coli*?

(b) If a sample of *E. coli* contains 5000 bacteria, how many will there be after 1 hour?

(c) If a sample of *E. coli* contains 1000 bacteria, how many will there be after 1 day?

4. The deer population of a national park was 250 deer 12 years ago. Today, there are 500 deer. Assuming the deer population has experienced exponential growth, write a relation representing the size of the deer population in the park. Use your relation to project the deer population in 25 years.

5. The relation $T = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$ can be used to determine the length of time, t , in hours, that milk of a certain fat content will remain fresh. T is the storage temperature, in degrees Celsius.

(a) What is the freshness half-life of milk?

(b) How long will milk keep fresh at 22°C?

(c) How long will milk keep fresh at 4°C?

Answers: 1. (a) 36 hours, (b) 15 000, (c) 30 000, two doubling periods in 72 hours, so bacteria doubled twice;
2. (a) 1 part per trillion, (b) 0.5 parts per trillion, (c) 0.00236 parts per trillion;
3. (a) 20 min, (b) 40 000, (c) 4.7×10^{24} ; 4. (a) $P = 250(2)^{\frac{t}{12}}$, (b) 2119;
5. (a) 10 hrs, (b) 31 hrs, (c) 56 hrs