

Worksheet 5-7: Explore Exponential Growth and Decay with Technology

Exponential Growth: $y = a(b)^x$, where y is the total amount, and x is number of changes over time, with an initial amount of a , and a growth factor of b when $b > 1$.

1. Animal Population

In a national park, a wolf population increased by a growth factor of 1.078 per year over a ten-year period, beginning in 1997. The formula $P = 124(1.078)^n$ modelled the wolf population after n years.

(a) Without graphing, state the wolf population in 1997. Explain how you get your answer.

(b) Use a graphing calculator to graph the relation. (Hint: Follow the steps below.)

Press **2nd** [STATPLOT]. Select **4:PlotsOff**. Press **ENTER**. (To clear scatter plots in RAM)

Press **Y=**. If necessary, clear all equations.

Type $124 \times 1.078 ^$, and then press **X,T,θ,n**. (Press **X,T,θ,n** for the variable x .)

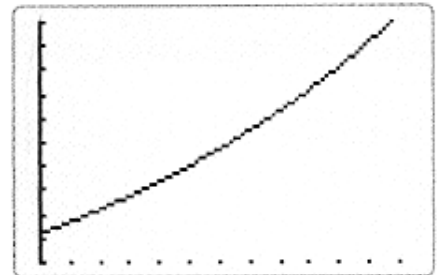
Press **WINDOW**. Use the window settings shown.

```

WINDOW
Xmin=0
Xmax=12
Xscl=1
Ymin=100
Ymax=300
Yscl=20
Xres=1
    
```

```

Plot1 Plot2 Plot3
Y1=124*1.078^X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
    
```



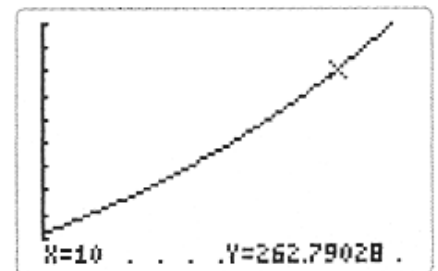
Press **GRAPH**.

(c) What was the wolf population in 2007?

Press **2nd** [CALC]. Select **1:value**.

Press **ENTER**, then enter 10 for X=. (Year 2007 is 10 years after 1997, $x = 10$ represents 2007)

Press **ENTER**.



Exponential Decay: $y = a(b)^x$, where y is the remaining amount, and x is number of changes over time, with an initial amount of a , and a decay factor of b when $0 > b > 1$.

2. Light Intensity

A sheet of translucent glass 1 mm thick reduces the intensity of the light passing through it. Light intensity is further reduced as more sheets of glass are placed together, as shown in the table.

Number of Glass Sheets	0	1	2	3	4	5	6	7	8
Light Intensity (%)	100	89.1	79.4	70.7	63.0	56.1	50.0	44.5	39.7

- (a) What is the decay factor for the relation?
- (b) What is the initial amount of light intensity?
- (c) Write the formula that models the above situation.
- (d) The reduction rate of a sheet of glass is the percent by which the light intensity is reduced by adding a sheet of glass to a viewing panel. What is the light intensity reduction rate of a single sheet of glass? Express your answer as a percent.

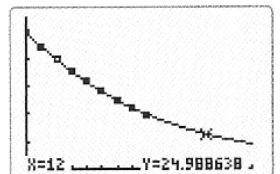
- (e) Use a graphing calculator to graph the relation. Refer to steps for Question #1 (b). Press **WINDOW**. Use the window settings shown.

```

WINDOW
Xmin=0
Xmax=15
Xscl=1
Ymin=0
Ymax=110
Yscl=2
Xres=1
    
```

- (f) How many sheets of glass are needed to reduce the light intensity by one half?

- (g) How many sheets of glass are needed to reduce the light intensity to about 25%? Press **TRACE**. Use the cursor keys to move the point shown on the graph until the value of **Y** is as close as possible to 25.



3. Cells in a culture are growing by a factor of 3.45 per day. The number of cells in the culture, N , can be estimated using the formula $N = 1000(3.45)^d$, where d is the number of days.

(a) Use a graphing calculator to plot the graph of this relation.

Press **2nd** [STATPLOT]. Select **4:PlotsOff**. Press **ENTER**. (To clear scatter plots in RAM)

Press **Y=**. If necessary, clear all equations.

Type **1000** **x** **3.45** **^**, and then press **x,T,θ,n**. (Press **x,T,θ,n** for the variable x .)

Press **ZOOM**. Select **“ZoomFit”**.

(b) What is the growth factor of the cells in the culture?

(c) How many cells does this culture begin with?

(d) How many cells would there be after 1 day?

Press **2nd** [CALC]. Select **1:value**.

(e) How many cells would there be after 5 days?

Press **2nd** [CALC]. Select **1:value**.

4. A deer population is declining by 2.2% per year. The population can be modelled using the formula $P = 240(0.978)^n$, where P is the population after n years.

(a) Use a graphing calculator to plot the graph of this relation.

Press **2nd** [STATPLOT]. Select **4:PlotsOff**. Press **ENTER**. (To clear scatter plots in RAM)

Press **Y=**. If necessary, clear all equations.

Type in the equation. Press **X,T,θ,n** for x .

Press **ZOOM**. Select “ZoomFit”.

(b) What is the current deer population?

(c) What is the declining rate of the deer population per year? Express your answer as a percent.

(d) What is the decay factor?

(e) What will be the expected deer population after 8 years?

Press **2nd** [CALC]. Select **1:value**.

(f) How long does it take to reduce the deer population by one half?

Press **TRACE**. Use the cursor keys to move the point shown on the graph until the value of **Y** is as close as possible to 120.

Answers: 1. (a) 124, given by the equation I is the initial amount, (c) 263;

2. (a) 0.891 (calculate common ratio of *dependent* variables), (b) 100, (c) $y = 100(0.891)^x$, (d) 10.9%,
(f) 6, (g) 12;

3. (b) 3.45, (c) 1000, (d) 3450, (e) 488 760;

4. (b) 240, (c) 2.2%, (d) 0.978, (e) 201, (f) 32 years.