Worksheet 1-6: The Cosine Law (Non-Right Triangles)

The cosine law can be used to solve a non-right triangle when given:

- (a) the measures of two sides and the "contained" angle
- (b) the measures of **three sides**

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$

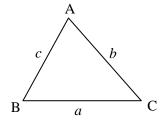
$$cos A = \frac{b^{2} + c^{2} - a^{2}}{2bc}$$

$$b^{2} = a^{2} + c^{2} - 2ac \cos B$$

$$cos B = \frac{a^{2} + c^{2} - b^{2}}{2ac}$$

$$c^{2} = a^{2} + b^{2} - 2ab \cos C$$

$$cos C = \frac{a^{2} + b^{2} - c^{2}}{2ab}$$



for side lengths

for angles

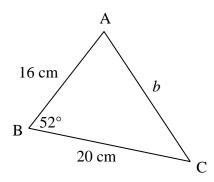
When do we need to use it?

We use it when the given information is **insufficient** for us to use sine law to solve the acute triangle.

For Rule (a) above, why do we need the contained angle to use the cosine law?

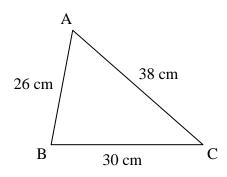
Find the Measure of a Side:

1. Find the measure of side b to the nearest tenth of a centimetre.

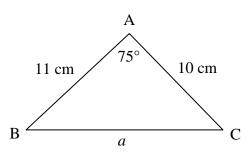


Find the Measure of an Angle:

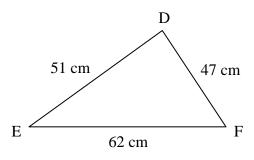
2. Find the measure of $\angle A$ to the nearest degree.



- 3. Solve each triangle, side lengths to the nearest tenth of a centimetre and angle to the nearest degree.
- (a) When Two Sides and the Contained Angle are Given

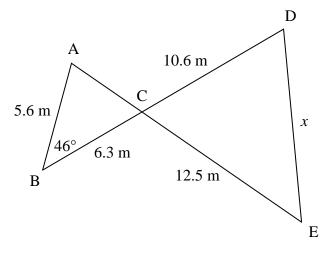


(b) When Three Side Lengths are Given



4. For $\triangle RST$, $\angle R = 72^{\circ}$, s = 12 m, and t = 10 m. Solve the triangle, to the nearest degree and metre.

5. Solve for *x* to the nearest tenth of a metre.



6. What information about a triangle do you need to know to use the cosine law?

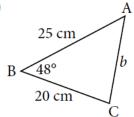
Answers: 1. 16.2 cm; **2.** 52°; **3.** (a) a = 12.8 cm, $\angle B = 49^{\circ}$, $\angle C = 56^{\circ}$, (b) $\angle D = 78^{\circ}$, $\angle E = 48^{\circ}$, $\angle F = 54^{\circ}$;

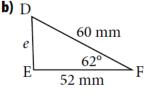
- **4.** r = 13 m, $\angle S = 61^{\circ}$, $\angle T = 47^{\circ}$; **5.** x = 11.5 m (AC = 4.7 m, $\angle ACB = 59^{\circ}$);
- 6. measures of all three sides or measures of two sides and the contained angle (angle in between).

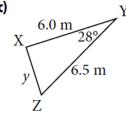
Practise

1. Find the measure of the unknown side.

a)

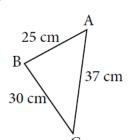




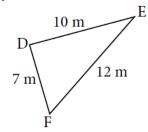


2. Find the measure of the unknown angle as indicated.

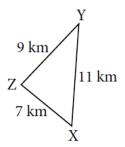
a) Find $\angle A$.



b) Find $\angle D$.



c) Find $\angle X$.



- **3.** Find the measures of the unknown sides and angles in \triangle ABC, given $\angle A = 32^{\circ}$, b = 25.5 m, and c = 22.5 m. Round side lengths to the nearest tenth of a metre and angles to the nearest degree.
- **4.** Find the measures of the unknown angles in \triangle ABC, given a = 14 m, b = 15 m, and c = 8 m. Round angles to the nearest degree.

Answers

- **1. a)** 18.9 cm
- **b)** 58.1 mm
- **c)** 3.1 m

- **2. a)** 53.7°
- **b)** 88.0°
- c) 54.7°
- **3.** $\angle B = 86^{\circ}, \angle C = 62^{\circ}, a = 13.5 \text{ m}$
- **4.** $\angle A = 67^{\circ}, \angle B = 81^{\circ}, \angle C = 32^{\circ}$